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The combination of input-output analysis and linear programming for water resource management: An application to Northwest Iowa

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CHAPTER I. INTRODUCTION

Problem Statements

Scarcity is a fact of life and the habitual condition of civilized society. Goods have never been available in such abundance as to exhaust all human wants. Natural resources have never been and will never be unlimited. What had been a somewhat remote controversy among the committed few over the limits of growth due to natural resource scarcity was brought home forcefully during the energy crisis of 1973-1974 and its current revival (69, p. xvii). Water, once regarded as a free good, is no exception. The supply of water for households, agriculture, and industry in the United States is a real and growing problem (80, p. 1).

Although Iowa has been endowed with ample average rainfall, growing water demand coupled with variations in annual rainfall has brought about mounting concern about water availability for Iowa's future economic growth (79, p. 143). It is gradually recognized that planning of and control over water resources to assure their optimum uses and adequate supplies are of critical importance to continued expansion of the state's economy. Accordingly, it becomes necessary to make an overall economic evaluation of the water supply and demand situation at both state and regional levels in order to establish the basis for decisions involving long-run economic planning of water.

Typically, a natural resource is put to a wide range of uses; almost all economic activities require water. To complicate natural resources management, these uses are interrelated with each other by a web of interdependences among them. For instance, agriculture uses water and also such inputs as fertilizers, pesticides, farm equipment, and energy, which are produced by other sectors. Production of these intermediate products also requires water, so that, if these intermediate products cannot be adequately produced because of inadequate water supplies, agricultural products cannot be adequately produced either.

Due to the interdependences among uses of a natural resource, once a shortage of the resource arises, its impact is not limited to a small segment of the economy, but tends to permeate the entire economy. Hence, no one use of a resource can be singled out for effective control and planning. It is necessary to treat all uses of the resource, or resource uses in the context of this study, simultaneously. Input-output analysis is frequently used to deal with the interdependences among resource uses. This approach was adopted by Barnard and Dent in their water study for Iowa (5).

However, emphasis on the interdependence dimension of the resource uses should not lose sight of a resource allocation dimension of the resource uses. Uses of a natural resource are not only interdependent, but also competitive. One committed use of a resource can exclude other uses. This requires purposeful allocation of resources among competing uses. What is needed for an effective overall evaluation of the demand and supply situation of a natural resource is an integrated view of the economy as a whole through a comprehensive model with natural resources as an integral part and with both the interdependent and competitive dimensions combined.

Study Objectives

The general objectives of this study are (1) to develop an operational model integrating both competitive and interdependent dimensions of water uses, (2) to apply it to Northwest Iowa, and (3) to suggest a revised model for the future application to state and regions. More specifically, the study objectives are:

1. to determine the level of final demands to support projected population and economic growth;
2. to estimate production and water requirements to satisfy the final demands;
3. to derive the shadow prices of water in alternative uses;
4. to draw implications on water reallocation; and
5. to make suggestions for further research needs.

Procedures of Model Development and Application to Study Area

The model to be developed is the combination of linear programming and input-output analysis. It consists of the objective function in terms of maximizing income, the input-output system, and the resource constraints. Two versions of the model are presented. In the first version, the input-output system is an open system which consists of production activities alone. Such a system will be called the basic input-output system. This first version was put to an application in this study.

The input-output system in the second version of the model is an extension of the basic input-output system to incorporate the income consumption linkage and resource use. Such a system will be called the extended input-output system. This version of the model is suggested for future application to obtain more accurate estimates of production and resource requirements and, more importantly, to conduct impact analysis in conjunction with possible multiple objectives.

The location selected for an application of the model is the 12-county area in Northwest Iowa (see Figure 1, p. 30). This region is chosen primarily because of availability of water supply data. This region is known to have more water-related problems than any other regions of Iowa (64). Annual rainfall is the lowest in Iowa, ranging from 25 to 28 inches per year. Ground water is available, but not in sufficient quantities for many uses. As a result, this region has received much attention. Colbert conducted a productivity analysis of irrigation water in Northwest Iowa (15). Babula made a detailed study of farm profitability of irrigation in this region (2). Rossmiller developed a goal programming model for comprehensive water and land management of the region (64). The two former studies concentrated on the irrigation problem as it relates to crop productivity. Rossmiller's work does not deal with the intersectoral relationships in water uses.

The objective function of the applied model was addressed in terms of maximization of the region's net income. It was assumed that water cannot be transported from one subregion (county) to another subregion (county)[1]. Therefore, water and land constraints were imposed on each subregion in terms of availability of them for each subregion.

Also it was assumed that all water available from the region is of homogeneous quality so that demand for and supply of water were not differentiated in terms of water quality. In other words, application of the model was concerned with water quantity alone. However, instead of being homogeneous, water is extremely heterogeneous in terms of its properties, its technologically permitted uses, and its economically demanded uses (80, p. 6). The total quantity of water may be abundant, but we may not have available sufficient water of a particular quality to satisfy a particular use demand. Therefore, taking water quality variations into account would significantly modify the results of this study's application. In the light of the importance of the water quality problem, a further discussion of this problem will be presented in the last chapter under further research needs.

Since no input-output table focused on this region is available, it was assumed that the production structure of this region is similar to the overall production structure of the state as embodied in the input-output table of the state. Thus, the technical coefficient matrix

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[1]In Iowa, any transfer of water beyond exempted amounts by uses from whatever sources to whatever locations for whatever purposes must first receive the approval of the state in the form of a permit from the Iowa Natural Resources Council (64, pp. 48-51).

and value added coefficients (more precisely, income coefficients) of the state become applicable to this region. That is, the Northwest Iowa economy was treated as a miniature of the whole Iowa economy. To the region's industrial sectors except for the crop production sector, the water coefficients estimated by Barnard and Dent for the state were applied (5, pp. 75-76). The water coefficient of crop production was estimated from irrigation water requirement data of the region. Estimation of the water coefficients of final water uses was based on the data provided by Rossmiller for the region (64). Estimates of final demands of the region, derived on the basis of the income data of the region, are an important part of the data series used in this study, because they determine the total water requirement of the region.

The Iowa economy is expected to continue its expansion at modest growth rates (5, pp. 1-24). Population is expected to grow at an annual pace of 0.24%, employment at 0.51%, total income at 2.85%, and per capita income at 2.94%, respectively. Based on these growth rates, the State of Iowa has made long-term projections of population and economic growth of Iowa to the year 2020 (5, 39). Considering some regional variations, it has also made long-term projections of regional population and economic trends. Since Northwest Iowa is known to have been endowed with less average rainfall than any other regions of Iowa, the primary purpose of the application is to investigate whether or not the water resources of Northwest Iowa can support the region's population and economic growth as projected by the State of Iowa to the year 2020, given the water use rates of 1967 and given the production structure and inter-sectoral relations as embodied in the 1967 input-output table of Iowa[1]. The base year for economic activities was set at 1975[2].

Since the applied model employs the basic input-output system, the underlying assumption is that a change in production creates a change in income, but this resulting change in income has no feed-back effect through the income consumption linkage on production and, hence, on resource uses. This effect will be simply called the income effect throughout this study. An increase in production entails an increase in income. This increase in income induces additional consumption which in turn induces additional production and resource uses, thereby increasing income again. Thus, production, resource use, income, and consumption form a cause and effect circle. Emphasizing this income effect on water uses, Timmons (78, p. 1245) states that

> Increasing demand upon available water supplies are unmistakable. These demands are growing at an increasing rate stemming (1) from our growing population and (2) particularly from our increasing per capita use of water which is about twice as rapid as our rate of population growth.

Many studies report an increasing per capita water consumption stemming from growing affluence. For example, in California, per capita household water consumption stood at 140 gallons per day in 1950, but rose to an annual average of 172 gallons per day for the period 1961-1965 (55, p. 124).

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[1]Some of the coefficients were updated (see p. 37).

[2]The price of corn was set at that of 1978.

In input-output analysis, the significance of the income effect is frequently quantified in terms of the so-called type II multiplier (51, 58, 62, 84). The type II income and output multiplier measure the effects of a change in autonomous spending (final demand) on the level of income and production when the income effect is taken into account. Since the type I income and output multiplier do not take the income effect into account, the differences between the type II and type I multipliers indicate the size of the income effect.

In his input-output study on the Iowa economy, Barnard estimated the two types of the output multiplier for 77 industrial sectors in Iowa (4, p. 55)[1]. According to the result, the difference between the type II output multiplier and the type I output multiplier ranges from 0.06 for the office computing and accounting machinery industry to 2.0 for the electric lighting and wiring equipment industry, implying that taking the income effect into account leads to 6 to 200 percent higher production estimates than those obtained when such income effect is left out of account. Taking the average difference at two, a cursory approximation is that the total water requirement would be doubled in Iowa when the income effect is taken into account. If so, this would significantly modify the results obtained from the applied model.

Even though this approximation is very tentative, the difference between the two types of multipliers becomes an important consideration

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[1]Barnard used "simple" for the type I and "total" for the type II.

in overall estimation of production and resource requirements when the income effect is fully counted in. In fact, as will be proved later, the ratio of the two types of the income multiplier is none other than the Keynesian multiplier. That is, the type II income multiplier is a constant multiple of the type I income multiplier and the constant is the Keynesian multiplier.

Therefore, accuracy in estimation of production and resource requirements of an economy hinges much on estimation of the Keynesian multiplier especially when the income effect is significant. The Keynesian multiplier as derived in this study is the one reflecting (1) interdependences among producing sectors of the economy via the flow of intermediate goods and (2) limits of resource supplies available for the economy. In order to distinguish this Keynesian multiplier from the Keynesian multiplier of the orthodox Keynesian macro-model, it will be termed the input-output Keynesian multiplier labelled by M, because making several assumptions readily reduces it to the Keynesian multiplier of the orthodox Keynesian model. When a shortage of a particular resource dampens the multiplier effect of autonomous spending, the resulting modified input-output Keynesian multiplier will be called the resource constrained Keynesian multiplier, denoted by $\hat{M}$.

Development of an extended model (the second version of the model) begins with an extension of the basic input-output system. The consumption function is incorporated into the system through income. Two types of consumption expenditures are distinguished: consumption expenditures on produced goods and services and those on non-produced goods and services (i.e. resource inputs). The resulting extended input-output system provides the input-output Keynesian multiplier and clarification of various multiplier processes. One feature of this extended input-output system is that it is expressed in terms of the Leontief matrix of the basic input-output system. As a result, the solutions for production, resource employment, and income as derived from the extended input-output system involve the Leontief inverse of the basic input-output system. Since this Leontief inverse matrix is available from any input-output table, to obtain such solutions does not require any matrix inversion process.

Combining the extended input-output system with the resource supply constraints leads to the linear programming model to be suggested for future application.

Based on the extended model, an impact analysis or a postoptimality type of analysis is presented. The analysis is intended to explain how a resource shortage affects the level of income, production, and resource employment of the economy. This idea is closely related to the concept of shadow price, because a shadow price of a resource constraint indicates the impact of a change in the constraint on the objective to be pursued. The resource constrained Keynesian multipliers are derived from the impact analysis.

Besides the objective of maximization of income (efficiency objective), natural resource management frequently involves multiple objectives, for example, income distribution, national security, environmental quality, balance of payment, etc. (73, p. 40). One common approach to deal with the multiplicity of objectives is to maximize one objective (usually the efficiency objective) subject to other stated objectives (34, p. 223). The impact analysis presented by this study is also intended to explain how such multiplicity of objectives constrains resource allocation decisions, especially when a resource shortage surges up.

The shadow price is always relative to the objective to be pursued. A different objective leads to a different set of shadow prices. Thus, the operational meaning of the shadow price can be defined precisely only with reference to a particular objective. The shadow price as formulated from the impact analysis is expressed as a function with the input-output Keynesian multiplier and resource allocation decisions as arguments. Since the income effect can significantly alter the overall resource supply and demand situation and the input-output Keynesian multiplier reflects such income effect, the size of a shadow price is accordingly influenced by the size of the input-output Keynesian multiplier. The resource allocation decisions in what may be called the shadow price equation can be subject to multiple objectives.

Organization of Report

Chapter I introduces the problem area and outlines the specific objectives of the research covered in this study. Chapter II develops the model to be applied. Chapter III is devoted to application of the model and presents results of application. Chapter IV develops the extended model for future application. An extension of the basic input-output system and multiplier analysis is presented in the first section of Chapter IV. The second section of this chapter includes formulation of the linear programming model incorporating the income effect. Impact analysis is presented in the last section of the chapter.

Concluding remarks and further research needs are presented in Chapter V. Discussion of further research needs includes operational procedures for the future application of the extended model to Northwest Iowa and to the entire state.

CHAPTER II. FORMULATION OF THE MODEL

Since the model to be applied includes the basic input-output system, a brief review of this system is presented. The input-output system considered in the applied model is the open system which treats final demand as exogenous. The closed system which treats all or a part of final demand as endogenous is discussed when developing the revised model in Chapter IV. The review is summarized from Dorfman, Samuelson, and Solow (17) and Chenery and Clark (12), adopting their notation and their convention of model presentation. This is followed by combining the basic input-output system and resource constraints to construct the linear programming model. In the initial stage, the model is addressed in general terms in order to facilitate clarification of the nature of the model and also conjoint relationships between linear programming and input-output analysis. This generalization is necessary particularly because of wide variations in the method of combining linear programming and input-output analysis and resulting possible confusions in model interpretations[1]. After concrete specification of the model with particular reference made to the case-study area is presented, the conjoint relationships between linear programming and input-output analysis are discussed, with the linear programming model addressed in general terms.

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[1]See Dorfman, Samuelson, and Solow (17, p. 212), Chapter IV of Chenery and Clark (12), Heesterman (33), Schluter and Dyer (68), and also Brink and McCarl (9).

The Basic Input-output System

Consider an economy consisting of n producing sectors where the following three assumptions hold (12, p. 33):

1. each commodity (or groups of commodities) is produced by a single sector[1];
2. the inputs purchased by each sector are a function of the level of output of that sector; and
3. the total effect of carrying out several types of production is the sum of the separate effects[2].

Let

x_i = total production of sector i;

x_{ij} = amount of an intermediate input produced by sector i and used in sector j; and

f_i = final demand for a product produced by sector i = final output of sector i.

There is no fixed rule for including (or excluding) any specific economic activity in the final demand category. However, major final demand items usually include household consumption, government expenditures, exports, and autonomous investment. The input-output table of Iowa developed by Barnard treats these items under final demand (4, pp. 70-140).

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[1]The input-output table of Iowa developed by Barnard divides the Iowa economy into 77 industrial sectors with the first sector designated as the livestock production sector and the second sector as the crop production sector (4, p. 55).

[2]This is known as the additivity assumption which rules out external economies and diseconomies.

The input-output system basically hinges on two kinds of relationships (17, p. 230). First, the bookkeeping identity that the total output of any sector must be allocated as intermediate goods (x_{ij}'s) and final outputs (f_i's) as expressed in the following equations:

$$x_i = x_{i1} + x_{i2} + .. + x_{in} + f_i, \; i = 1, 2, \ldots, n \qquad (1)$$

Secondly, the technological relationship that purchases of intermediate inputs by any sector from any other sector depend, via the production function, on the level of output of the purchasing sector as expressed in the following equations:

$$x_j = F^j(x_{1j}, x_{2j}, \ldots, x_{nj}, x_{oj}), \; j = 1, 2, \ldots, n \qquad (2)$$

where F^j is assumed to be a homogeneous function of the first degree and x_{oj} represents the total use of a primary input in sector j.

Equation (1) can be reduced to a computationally manageable system by the assumption that each x_{ij} is a homogeneous function of output x_j, i.e.,

$$x_{ij} = a_{ij}x_j$$

where a_{ij} is called the technical coefficients. Therefore, the Equation (1) is rewritten as:

$$(I-A)x = f \qquad (3)$$

where $x = (x_1, x_2, \ldots, x_n)'$, $f = (f_1, f_2, \ldots, f_n)'$ and $A = ((a_{ij}))$. The matrix (I-A) is known as the Leontief matrix.

One of the most important applications of input-output analysis is to calculate the equilibrium output levels in each sector of the economy. Output is in equilibrium if it is just equal to the quantity demanded for all purposes: consumption, investment, inventories, exports, and so on. If this quantity demanded for all purposes is given and A is known, then the equilibrium output levels are calculated from

$$x = (I-A)^{-1} f \ . \qquad (4)$$

The inverse matrix $(I-A)^{-1}$ is referred to as the Leontief inverse. Each element e_{ij} of $(I-A)^{-1}$ indicates the total production directly and indirectly required from industrial sector i for each unit of delivery of industrial sector j to final demand. The vector x indicates the production requirements of the producing sectors to support exogenously specified final demands. Throughout this study, such x will be called the input-output solution.

The existence of the Leontief inverse is crucial to the existence of the input-output solution. For the existence of this inverse, the following theorem, which will be used several times in this study, is available (75, p. 392):

Theorem 1: Let $M = ((m_{ij}))$ be a (nxn) matrix with $m_{ij} \leq 0$ for $i \neq j$. Then the following four conditions are mutually equivalent.

(I) There exists a $x \geq 0$ such that $Mx > 0$ (i.e. for some $f > 0$, there exists a $x \geq 0$ such that $Mx = f$).

(II) For any $f \geq 0$, there exists an $x \geq 0$ such that $Mx = f$.

(III) The matrix M is non-singular and $M^{-1} \geq 0$.

(IV) All the successive principal minors of M are positive.

In other words,

$$m_{11} > 0, \quad \begin{vmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{vmatrix} > 0, \quad \ldots\ldots\ldots, \quad \begin{vmatrix} m_{11} & m_{12} & \cdots\cdots & m_{1n} \\ m_{21} & m_{22} & \cdots\cdots & m_{2n} \\ \vdots & \vdots & & \vdots \\ m_{n1} & m_{n2} & \cdots\cdots & m_{nn} \end{vmatrix} > 0$$

The condition (IV) is known as the Hawkin-Simon conditions.

Notice that off-diagonal elements of the Leontief matrix (I-A) are non-positive. When an input-output table is made for a particular year, positive x and f are actually observed, so that (I-A)x = f for that year. Hence, condition (1) of the above theorem is satisfied. It follows that there exists an inverse so that for any final demand vector f Equation (4) holds.

The input-output system is related to the national income account; final demands (f_i's) represent the output side of GNP, and primary input (x_{oj}'s) the factor cost side. The interindustry sales (x_{ij}'s) have no welfare significance. The primary inputs are the economy's only income earning inputs, and thus all value-added (GNP) is due to the sales of the primary inputs. The input-output system views the economy as a productive machine that uses up primary inputs and produces final outputs for consumption.

The Linear Programming Model

The basic structure of the model and interpretations

Suppose that income maximization for the region is the single goal of the region's economy. Income considered in this study is disposable income. Suppose that there are m different primary and natural resources the region can utilize[1]. Let

v_i = income generated per dollar of output produced by sector i

b_{ij} = amount of resource i required to produce one dollar output in sector j

r_i = total amount of resource i required for the region's economy

$\bar{r}_i$ = total amount of resource i available for the region's economy

$v' = (v_1, v_2, \ldots, v_n)$

$B_{ij} = ((b_{ij}))$ = mxn matrix of b_{ij}

$r' = (r_1, r_2, \ldots, r_m)$

$\bar{r}' = (\bar{r}_1, \bar{r}_2, \ldots, \bar{r}_m)$.

We will call v_i and b_{ij} the income coefficient and the resource coefficient, respectively. Let the k-th constraint to be on water aggregated over all water supply sources. Then, b_{kj} is the water coefficient of sector j.

Combining the basic input-output system, the objective function, and resource constraints leads to the following linear programming problem:

---

[1]The primary resource refers to such resources as labor and capital.

$$\text{Max } v'x$$

subject to

$$Bx \leq \bar{r} \tag{5}$$

$$(I-A)x \leq f \tag{6}$$

$$x \geq 0$$

i.e., maximizing income subject to resource availability $\bar{r}$ and to the exogeneously specified final demands f. Equation (5) says that the total demand for each resource must be less than or equal to the total supply of it. Because attainable production x is constrained by the resource availability, the exogenously specified final demands may not be achieved because of resource shortages. Therefore, Equation (6) indicates that the realized final demands supported by the given resource supplies (the left-hand side) are less than or equal to the exogenously specified final demands f. Let's denote the realized final demands by $\hat{f}$. Then, the resource requirements associated with $\hat{f}$ are given by

$$r = B(I-A)^{-1}\hat{f} \ . \tag{7}$$

The water requirement for $\hat{f}$ is given by

$$r_k = B_k'(I-A)^{-1}\hat{f} \tag{7b}$$

where B_k' is the k-th row of the resource coefficient matrix B. The water requirement as given by Equation (7b) represents the total requirement with interdependences among water uses taken into account. In the context of interdependences among water uses, the water requirement is frequently categorized under two types: the direct requirement and the indirect requirement (5, p. 77).

The direct requirement of a certain water use represents the amount of water directly required in producing one unit of an output of the use. In our model, b_{kj} represents the direct water requirement of sector j which is also called water coefficient[1]. For example, an estimated 14.4 gallons of water is directly required in producing one dollar of livestock products in Iowa (5, p. 75).

The indirect requirement stands for the amount of water indirectly required to produce other inputs that are used to produce one dollar value product of an output of the use. If the j-th column of the Leontief inverse is denoted by E_j, the indirect requirement is given by

$$B_k'E_j - b_{kj} \ .$$

For example, an estimated 23 gallons of water is indirectly required to produce intermediate inputs used in the livestock sector per dollar of livestock products in Iowa (5, p. 78), i.e., 1.6 times as much as the direct requirement is indirectly required. This implies that, for instance, to export one dollar value of livestock product or to deliver the same amount to households, a total of 37.4 gallons of water is required in production process of livestock products in Iowa. It follows that a projection for the livestock water requirement should be

[1]Lofting and McGauhey uses the direct requirement and the water coefficient interchangeably (44, p. 23), while the direct requirement as used by Barnard is the element of $B_k'(I-A)^{-1}$ corresponding to the sector. For example, the j-th element of $B_k'(I-A)^{-1}$ represents the direct requirement of sector j (5, p. 77).

based on the total requirement with the direct and indirect requirement combined (i.e., 37.4 gallons) rather than merely the direct requirement (i.e., 14.4 gallons). Since the linear programming model comprises the input-output system, the water requirement reported by the model is this total requirement.

Specification of the constraints

In applying the model, use was made of the 13 industrial sector input-output table of Iowa Barnard developed (4, p. 34). The first sector is the livestock agricultural sector and the second sector the crop production sector. Table 6 of the next chapter enumerates the 13 sectors of the input-output table. Hence, the part of the model for the input-output system consists of 13 equations (i.e., n=13) with the 13x13 technical coefficient matrix (i.e., A matrix).

The case-study area comprises 12 counties of Northwest Iowa. The name of each county is given in Table 1 of the next chapter. In specifying the resource constraints, the following assumptions are made:

1. There exists an upper limit on each county's water availability from a particular water supply source and it is not augmented by water transportation;
2. All water available in the region is of the same quality;
3. Water supply sources of each county are independent of each other[1];

[1]This may not be true in reality. For instance, tapping surficial aquifers may reduce adjacent stream flow.

4. All the other resources except for water and land do not constrain each county's economic growth[1];

5. Land availability constrains, if it does, crop production only;

6. The water coefficient of a particular producing sector except for the crop production sector is the same over all counties; and

7. Only corn production is irrigated[2].

Since the primary concern of the model application is with water availability for the region's economic growth, specification of the resource constraints is focused on water demand and supply. The following notation will be used (the superscript refers to county):

x_i^k = total production of industrial sector i in county k,

$i = 1, 2, \ldots, 13, \quad k = 1, 2, \ldots, 12;$

x_{2a}^k = non-irrigated corn production in bushel;

x_{2b}^k = irrigated corn production in bushel;

x_{2c}^k = production of other crops (in dollar);

w_i = water coefficient of industrial sector i [3];

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[1]Labor supplies may constitute an important constraint. However, the economic growth projections made by the State of Iowa for this region have already taken labor supplies into account (5, pp. 1-13).

[2]The other crops, mainly soybeans, are known to be more tolerant of drought condition, except for sandy soils (64, p. 374).

[3]To avoid clutter, the water coefficient is labelled by w_i rather than b_{ki} as in the previous section.

w_2^k = irrigation water requirement of corn per bushel;

W^k = total water use of county k;

f_w^k = final use of water;

l_a^k = land requirement of non-irrigated corn production per bushel;

l_b^k = land requirement of irrigated corn production per bushel;

L_c^k = total land area available for crop production;

L_r^k = total land area available for irrigation.

Water supply data used in this study are based on the preliminary report on availability of water resources of Northwest Iowa prepared by the Iowa State Water Resources Research Institute. The report identifies seven water supply sources in Northwest Iowa. Using the notation of the report, they are:

GW_1 = the northwestern bedrock aquifer system that consists primarily of the Dakota Sandstone formation;

GW_2 = the surficial alluvial aquifer associated with the Missouri River flood plain;

GW_3 = other surficial aquifers including the one associated with the Big Sioux River;

SW_1 = the natural streamflow in the interior streams;

SW_2 = Missouri River;

SW_3 = Big Sioux River;

SW_4 = reservoir storage which augments natural streamflow.

As a result of this breakdown, there are three ground water sources and four surface water sources available for the region[1]. Let

GW_i^k = amount of water used from ground water supply source i;

SW_i^k = amount of water used from surface water supply source i;

$\overline{GW}_i^k$ = total supply of water available from ground water supply source i;

$\overline{SW}_i^k$ = total supply of water available from surface water supply source i.

Since individual water and land availabilities are imposed on each county, each county's production activities are constrained by its own water and land availability. The total amount of water used in county k is the sum of the irrigation water for corn production, the amount of water used by the other industrial activities, and final water uses in the county. Hence,

$$W^k = w_1x_1^k + w_2^kx_{2b}^k + w_3x_3^k + .. + w_{13}x_{13}^k + f_w^k, \ k = 1, \ldots, 12 \qquad (8)$$

The total amount of water used in county k consists of water supplied from each source. Hence,

$$W^k = GW_1^k + GW_2^k + GW_3^k + SW_1^k + \ldots + SW_4^k, \ k = 1, \ldots, 12 \qquad (9)$$

The amount of water that can be supplied from each water supply source is limited by the availability from each source:

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[1]Not all seven water supply sources are available for each county (see Table 16).

$$GW_i^k \le \overline{GW}_i^k \ , \quad k = 1, \ldots, 12, \quad i = 1, 2, 3 \tag{10}$$

$$SW_i^k \le \overline{SW}_i^k \ , \quad k = 1, \ldots, 12, \quad i = 1, 2, 3, 4 \tag{11}$$

Irrigation is limited by availability of land for irrigation. Corn production is related to land through the land coefficient. Therefore, we have

$$l_b^k x_{2b}^k \le L_r^k \ , \quad k = 1, \ldots, 12 \tag{12}$$

The total corn production is limited by availability of land for corn production. Therefore,

$$l_b^k x_{2b}^k + l_a^k x_{2a}^k \le L_c^k \ , \quad k = 1, \ldots, 12 \tag{13}$$

Crop production consists of three activities; irrigated corn production, non-irrigated corn production, and production of other crops. Hence,

$$x_2^k = p^c(x_{2a}^k + x_{2b}^k) + x_{2c}^k \ , \quad k = 1, \ldots, 12 \tag{14}$$

where p^c is the price of corn per bushel. Production of each product by each county should add up to the total regional production of the product:

$$x_i = x_i^1 + x_i^2 + \ldots, + x_i^{12} \ , \quad i = 1, \ldots, 13 \tag{15}$$

In the light of overall growth in every economic sector of Northwest Iowa as projected by the State of Iowa (see next section), it is assumed that each county's production of each product in the year

2020 be no less than that of 1975. Therefore,

$$x_i^k \geq \bar{x}_i^k , \quad k = 1, \ldots, 12, \quad i = 1, \ldots, 13 \tag{16}$$

where $\bar{x}_i^k$ is the production level in 1975 of industrial sector i in county k.

In the input-output system, the costs of every production activity should be addressed in terms of intermediate inputs and primary and natural resources. The water for most practical uses is in fact the produced water, produced from natural water. Water production involves pumping, treating, and delivering. Because of inadequate data on water production costs in terms of intermediate inputs and resources required in the process, an ad hoc measure is taken by this study to put monetary water supply costs directly into the objective function. In application, the objective function presented in the previous section is modified as follows:

$$\max v'x - \sum_{i}^{7} \sum_{k}^{12} c_i^k \tag{17}$$

where c_i^k stands for the cost of supplying water from water supply source i of county k.

Thus, the applied model consists of a total of 317 equations; the objective function of Equation (17), the basic input-output system of Equation (3), the resource constraints of Equations (8) - (13), definitional Equations (14) and (15), and the minimum production requirements of Equation (16).

Conjoint Relationships Between Linear Programming and Input-output Analysis

Even though the exogenously specified final demands f may not be achieved due to the resource constraints, it is simple to show that a solution for x that completely satisfies the given level of final demands is also the solution that potentially maximizes income. In other words, the input-output solution leads to a potential maximum income, even though it may not be feasible with respect to resource availability. Denoted by x^*, the input-output solution is characterized by

$$x^* = (I-A)^{-1}f.$$

Suppose that $\hat{x}$ is a linear programming solution which is attainable from the given resource supplies. Then,

$$v'x^* - v'\hat{x} = v'(x^* - \hat{x}) = v'[(I-A)^{-1}f - \hat{x}] .$$

Since $\hat{x}$ is feasible,

$$(I-A)\hat{x} \leq f$$

But $(I-A)^{-1} \geq 0$ by Theorem 1. Hence,

$$\hat{x} \leq (I-A)^{-1}f$$

i.e.,

$$(I-A)^{-1}f - \hat{x} \geq 0 .$$

Therefore,

$$v'x^* \geq v'\hat{x} \ . \quad \text{QED.}$$

The total resource requirements associated with the input-output solution x*, denoted by r*, is given by

$$r^* = Bx^* = B(I-A)^{-1}f \ . \tag{18}$$

Depending on the resource availability,

$$r^* \gtreqless \bar{r} \ .$$

If $r^* > \bar{r}$, this implies that the input-output solution is not sustainable with respect to the resource availability and accordingly the given bill of final demand cannot be attainable due to resource constraints.

Since input-output analysis that consists of the basic input-output system does not explicitly deal with the supply side of resources, using the analysis amounts to assuming implicitly that there exist resource supplies sufficient enough to cover its resulting solutions. That is, input-output analysis determines only the demand side of resources which may not be feasible with respect to availability of them. Since the above formulated linear programming model comprises the input-output system and the resource constraints, the model reports (1) the effective resource demand checked by resource availability but integrating both the direct and indirect requirements arising from interdependences among resource uses and (2) the shadow price arising from competition among resource demands over fixed resource supplies.

CHAPTER III. APPLICATION OF THE MODEL TO NORTHWEST IOWA

The case-study location selected for an application of the model is the 12-county area of Northwest Iowa shown in Figure 1. This region is bounded on the north by Minnesota and the western border is formed by the Missouri and Big Sioux Rivers which separate Iowa from Nebraska and South Dakota.

Table 1 lists the names of the counties of the study area and describes the status of each county in the region in terms of population and total income in 1975. The total population of the region was 295,614 people in 1975, which is about 10 percent of the total population of the state. The region's total income, 1,043 million dollars, was about 7.5 percent of the total income of the state in 1975. Woodbury County, whose western boundary is bordered by the Missouri River, is by far the largest county in the region in terms of population and income. This county harbors 36.8 percent of the region's total population and accounts for 36 percent of the region's total income generated in 1975. Sioux City, the largest town in the region, is located at the northwestern corner of Woodbury County. Only two cities in the region have a population in excess of 10,000; Sioux City with 85,925 and Spencer, in Clay County, with 10,278 (64, p. 281). The second largest county in the region in terms of population and income is Sioux County on the border of the Big Sioux River and the smallest county is Ida.

Table 2 describes income sources of each county in 1975. This shows that agriculture is the major industry in all counties except for Woodbury County where agriculture is a minor industry.

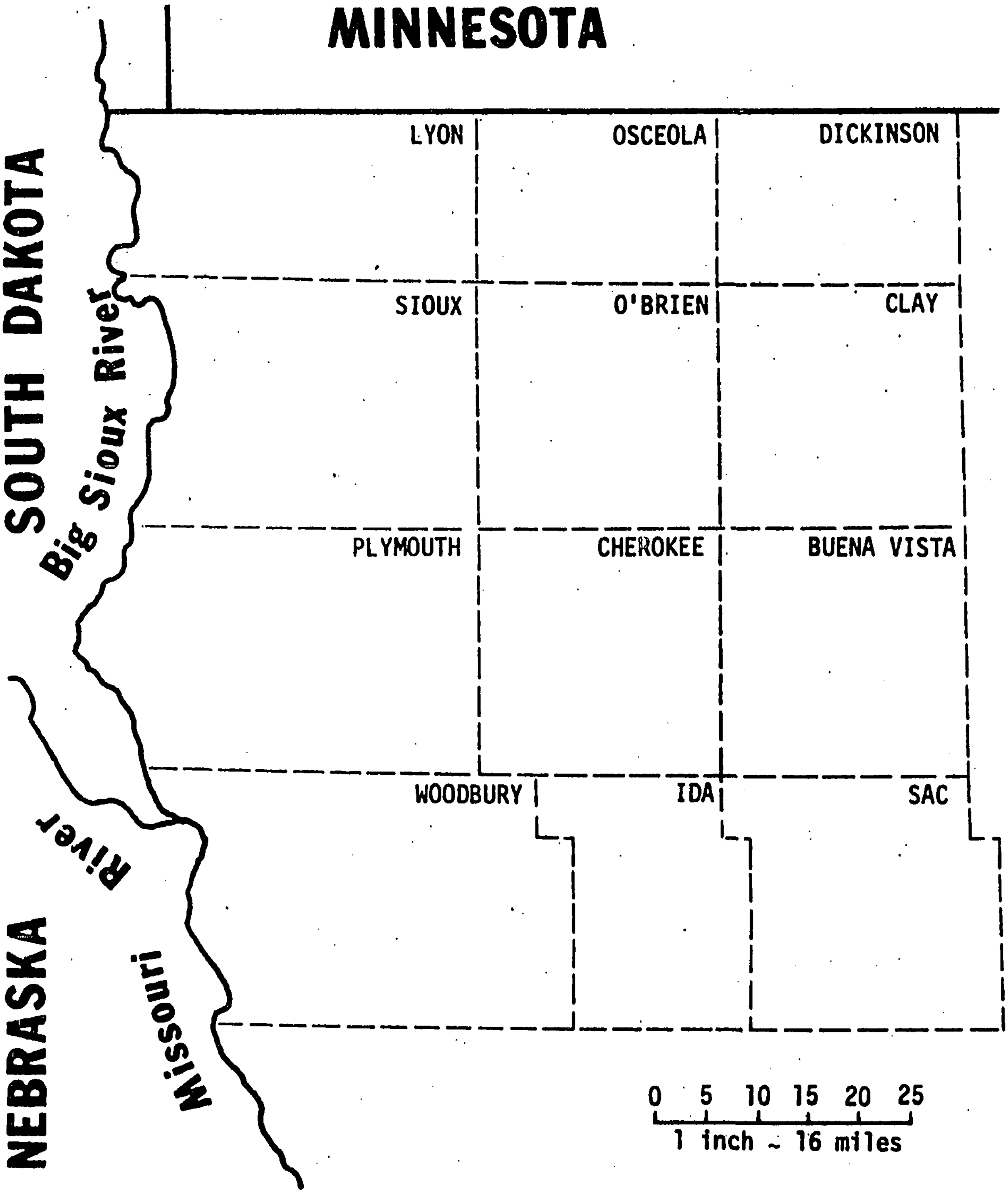

Figure 1. Study area in Northwest Iowa

Table 1. Population and total income of Northwest Iowa in 1975[a]

County	Population	Income ($ million)
Buena Vista	21,614	83
Cherokee	17,805	61
Clay	19,590	75
Dickinson	13,735	54
Ida	9,344	33
Lyon	12,705	44
O'Brien	17,375	57
Osceola	7,231	27
Plymouth	24,133	75
Sac	14,712	58
Sioux	28,600	100
Woodbury	108,770	376
Region	295,614	1,043

[a]Source (39).

Non-agricultural activities of the region are concentrated on Woodbury County and income generated from them are much larger than total income of any other county. As a result, for the region as a whole, manufacturing, trade, and services exceed agriculture as the sources of large income.

Table 2. Total income of Northwest Iowa by types of income sources in 1975[a]

County	Ag.[b]	Mi.[b]	Cn.[b]	Ma.[b]	Tn.[b]	Cm.[b]	Td.[b]	F.I.R.[b]	Sv.[b]
					($ million)				
Buena Vista	23	0	2	16	4	2	18	4	14
Cherokee	12	0	4	14	2	2	11	2	14
Clay	20	0	5	13	3	3	16	4	11
Dickinson	14	0	3	15	2	1	10	2	7
Ida	10	0	2	5	1	1	7	2	5
Lyon	16	0	2	7	1	1	8	3	6
O'Brien	13	0	2	10	2	3	13	2	12
Osceola	9	0	1	6	1	0	6	1	3
Plymouth	16	0	4	9	1	3	23	3	16
Sac	24	0	3	8	1	2	10	2	8

[a]Source (39).

[b]Ag. = agriculture; Mi. = mining; Cn. = construction; Ma. = manufacturing; Tn. = transportation and warehousing; Cm. = communication and utilities; Td. = trade; F.I.R. = finance, insurance, and real estate; Sv. = service.

Table 2. Continued

County	Ag.	Mi.	Cn.	Ma.	Tn.	Cm.	Td.	F.I.R.	Sv.
					($ million)				
Sioux	25	0	6	25	2	2	17	4	19
Woodbury	11	1	25	95	16	19	91	30	87
Region	193	1	59	224	36	39	230	59	202

Vigorous growth of non-agricultural production has been projected by the State of Iowa to the year 2020, as indicated by Table 3. Noticeable growth rates in terms of income are expected in manufacturing, communication, and service industry with 5.82, 9.28, and 7.36 percent per annum, respectively. A five percent annual growth rate is imposed on construction, utilities, and trade between 1975 and 2020. Agriculture is expected to grow at a moderate pace of 1.68 percent per annum. All these boil down to the annual overall growth rate of about five percent in the region's economy in terms of income between 1975 and 2020.

The projection of populations in Northwest Iowa made by the State of Iowa is detailed in Table 4. The total population of the region in the year 2020 is projected at 341,260, which is a 15.4 percent increase over the population of 1975. This amounts to a 0.34 percent annual growth in population. The decrease in rural farm population in all counties reflects the historic declines in agricultural employment.

Given these projections of economic and population growth of Northwest Iowa, the big step in application is to summarize them in terms of final demands and final uses of water of the region. Then application is geared to investigating, using the model developed in the previous chapter, whether or not the region can afford such growth projections with the region's endowment of the water resources which is known to be less favorable than in any other regions of Iowa, given

Table 3. Industrial growth of Northwest Iowa projected by the State of Iowa in terms of income between 1975 and 2020

Industry	1975 region[a] (1) ($ million)	1975 state[b] (2) ($ million)	1975 ratio (1)/(2) (%)	2020 region[a] ($ million)	Average annual growth rate from 1975 to 2020 (%)
Agriculture	193	1,635	11.7	307	1.68
Mining	1	50	2.0	6	2.78
Construction	59	786	7.5	176	4.94
Manufacturing	224	3,294	6.8	718	5.82
Transportation	36	481	7.5	107	3.72
Communication	21	209	10.0	112	9.28
Utilities	17	175	9.7	49	4.84
Trade	230	2,330	9.9	699	4.83
Finance, insurance, and real estate	59	727	8.1	246	7.36
Services	202	2,134	9.5	923	8.15
Region total	1,042			3,343	4.9

[a]Source (39).

[b]Source (5, pp. 23-28).

Table 4. Future total urban, rural farm and rural nonfarm populations in Northwest Iowa based on the projection made by the State of Iowa for the years 1980, 2000 and 2020[a]

	Total urban			Rural farm			Rural nonfarm		
	1980	2000	2020	1980	2000	2020	1980	2000	2020
Buena Vista	15,090	16,520	16,770	4,770	3,930	3,300	1,440	1,700	1,850
Cherokee	10,450	11,260	12,200	4,440	3,760	3,300	1,470	1,820	2,240
Clay	13,680	15,100	15,340	4,080	3,660	3,300	1,480	1,820	2,000
Dickinson	10,270	12,680	13,290	2,840	2,470	2,100	1,670	2,320	2,680
Ida	5,090	5,440	5,830	3,300	2,780	2,400	340	470	610
Lyon	6,870	8,930	11,630	5,400	4,260	3,410	750	1,080	1,520
O'Brien	12,140	13,950	14,930	4,850	3,960	3,300	1,010	1,410	1,740
Osceola	4,720	5,920	7,310	3,270	2,700	2,250	730	1,160	1,700
Plymouth	14,210	16,500	18,570	7,760	6,360	5,100	1,980	2,390	2,790
Sac	9,190	10,060	11,080	4,830	3,970	3,300	1,090	1,390	1,740
Sioux	20,420	25,620	28,770	8,310	6,290	4,500	1,390	1,850	2,130
Woodbury	99,190	110,630	118,380	6,320	5,560	5,120	3,090	3,950	4,750
Region	221,320	252,610	274,130	60,170	49,700	41,380	16,440	21,360	25,750

[a]Source (76).

1. the water use rates of 1967[1], and
2. the production structure and inter-sectoral relations as embodied in the 1967 input-output table of Iowa, with the base year set at 1975[2].

Economic Data Set

Coefficients

Four sets of coefficient data serve as inputs into the application of the model: technical coefficients (a_{ij}'s), income coefficients (v_i's), water coefficients (w_i's), and land coefficients (l_i's).

The technical coefficient matrix (matrix A) was derived from the 13x13 input-output table of Iowa made by Barnard (4, p. 34). Each column of the table was divided by gross production of the sector corresponding to the column to give a column of technical coefficients of the sector. The results are presented in Table 5. The income coefficient of a sector was formed by dividing disposable income accrued to that sector by gross production of the sector. The resulting income coefficients are listed in Table 6.

The pattern of utilizing water is unlike that for other natural resources. A given amount of water is not always completely 'consumed'

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[1]Where projected data were available, water use rates were updated to the year 2020. For example, water coefficients for crop production were based on yield data projected for 2020. Estimation of final water uses also made use of water use rates projected for 2020.

[2]The price of corn was set at that of 1978, which was $2.04 dollars per bushel (40).

Table 5. Technical coefficient matrix (matrix A)

1[a]	2	3	4	5	6	7	8	9	10	11	12	13
0.0936	0.0493	0.0000	0.4009	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0003
0.2030	0.0500	0.0041	0.0639	0.0012	0.0001	0.0001	0.0009	0.0001	0.0001	0.0013	0.0004	0.0007
0.0056	0.0094	0.0137	0.0021	0.0038	0.0019	0.0019	0.0201	0.0377	0.0371	0.0034	0.0515	0.0121
0.1051	0.0010	0.0020	0.1497	0.0073	0.0020	0.0027	0.0020	0.0020	0.0017	0.0024	0.0011	0.0085
0.0120	0.0692	0.0500	0.0414	0.2943	0.0420	0.0515	0.0515	0.0434	0.0080	0.0269	0.0120	0.0414
0.0002	0.0086	0.0000	0.0	0.0	0.0555	0.0	0.0	0.0	0.0	0.0	0.0	0.0004
0.0003	0.0011	0.0425	0.0006	0.0013	0.1614	0.1459	0.0295	0.0075	0.0039	0.0009	0.0001	0.0129
0.0017	0.0054	0.2687	0.0172	0.0160	0.1734	0.1463	0.2841	0.0179	0.0009	0.0029	0.0016	0.0240
0.0189	0.0083	0.0394	0.0361	0.0231	0.0206	0.0163	0.0264	0.0812	0.0165	0.0097	0.0025	0.0107
0.0040	0.0034	0.0072	0.0073	0.0118	0.0102	0.0123	0.0194	0.0165	0.1556	0.0278	0.0130	0.0292
0.0220	0.0221	0.0757	0.0299	0.0258	0.0421	0.0396	0.0273	0.0265	0.0074	0.0168	0.0057	0.0269
0.0202	0.1717	0.0139	0.0070	0.0226	0.0124	0.0164	0.0143	0.0400	0.0181	0.0700	0.0962	0.0404
0.0132	0.0340	0.0435	0.0214	0.0315	0.0320	0.0334	0.0236	0.0384	0.0453	0.0581	0.0308	0.0336

[a]1 = livestock; 2 = crop production; 3 = construction and mining; 4 = food and kindreds; 5 = other non-durables; 6 = farm machinery; 7 = other machinery; 8 = other durables; 9 = transport and warehousing; 10 = communication and utilities; 11 = trade; 12 = finance, insurance and real estate; 13 = services.

Table 6. Water and income coefficients of Iowa by types of industry[a]

Industry	Water coeff. (w_i) (gal. per $)	Income coeff. (v_i)
Agriculture		
livestock	14.4354	0.2526
crop production	4.1768	0.1563
Construction and mining	1.3084	0.3785
Manufacturing		
food and kindred prods.	1.2731	0.0913
other non-durables	4.6584	0.1888
farm machinery	0.7808	0.2653
other machinery	0.4042	0.3969
other durables	2.4211	0.2995
Transport	0.2746	0.4833
Communication and utilities	13.1994	0.2297
Trade	0.4583	0.5390
Finance, insurance and real estate	0.0316	0.1475
Service	0.7536	0.6768

[a]Computed from (4, p. 34) and (5, pp. 73-74).

by each of its uses but may be reused several times. In the wake of one use of water, a return flow or discharge frequently takes place. If this return flow does not create pollution problems and if it can be reused for other purposes, the relevant measurement of water use on which the concept of water productivity is based, is the reduction in available supply incurred from the use, termed consumptive use[1]. This study considers only consumptive water uses for industrial uses[2].

In their water study for Iowa, Barnard and Dent estimated consumptive water use of each industrial sector (5, p. 73). Dividing consumptive water use of each sector by gross production of the corresponding sector provides the water coefficients (Table 6).

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[1]Let

W_i = the amount of intake by use i

R_i = the fraction of return flow from use i

TW_i = the feasible total productive use of W_i

f_i = value of marginal productivity of water in use i.

Then,

$$TW_i = W_i + R_iW_i + R_{i+1}R_iW_i + R_{i+2}R_{i+1}R_iW_i + \cdots\cdots$$

The total value of marginal productivity of water taken by use i denoted by $TVMP_i$, can be expressed as

$$TVMP_i = f_i + f_{i+1}R_i + f_{i+2}R_iR_{i+1} + \cdots\cdots$$

Allocation efficiency requires (31, pp. 8-12)

$$TVMP_i = TVMP_j = TVMP_k = \cdots\cdots$$

[2]Due to data availability, final water uses (non-industrial water uses) are considered in terms of withdrawal (see next section).

According to the results, the livestock sector has the highest water requirement per dollar of output. It is followed by the communication and utility sector. The high water requirement in this sector is primarily due to utility production. According to Barnard and Dent, the utility sector is by far the largest water user in Iowa in terms of gross water use with 1,364 gallons per dollar of output. A large amount of discharge after use moderates consumptive water use of this sector to the level of 19 gallons per dollar of output (5, p. 76).

The water coefficient of crop production as estimated by Barnard and Dent is fairly modest with about 4 gallons per dollar of overall crop production. For example, in California where much of crop production depends on irrigation, the water coefficient of cotton production is reported at 2,986.4 gallons per dollar and that of other crop production at 2,251.3 gallons per dollar (44, p. 22)[1]. The low figure of the water coefficient of crop production estimated by Barnard and Dent is based on 1.094 acre-feet of water applied per irrigated acre which is the average value for the whole state where irrigation is only supplemental for crop production. Historically, by far the greatest use of water for irrigation has been in western and northwestern Iowa (29, p. 1). Halberg, Koch and Horick concluded in their report that this trend would continue in the future (29, p. 45). Table 7 demonstrates a greater interest in irrigation in western and northwestern Iowa than in other regions.

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[1]In terms of acre-feet, the numbers are 9,165 and 6,909 acre-feet, respectively. One acre-feet of water is equivalent to 325,851 gallons (37, p. 8).

The water coefficient of crop production quoted in Table 6 from Barnard and Dent needs to be modified to take into account the expected increase in irrigation in the region. Following Rossmiller (see the last section of Chapter II), it was assumed that only corn production is irrigated. Estimation of water coefficients of corn production is based on corn yields and on irrigation water requirements for corn.

Table 8 reproduces average irrigation water requirements of corn per acre for various return periods estimated by Rossmiller. An eleven-inch requirement for a 2 year return period means that eleven inches of irrigation water are required per acre in every two years. It is only twice in 100 years that the weather is so dry as to require 19.1 inches of irrigation water per acre for corn production. Calculating the average annual irrigation water requirement by using each return period as weight gives 11.16 inches per acre per year[1]. Tables 9 and 10 show fairly wide variations in projected corn yields among counties, between years, and between irrigation and non-irrigation. The non-irrigated corn yield projected for 2020 ranges from 164 bushels per acre of Lyon County to 201 bushels of Ida County. Substantial increases in yields are expected over years. Between 1980 and 2020, corn yields are expected to rise from the range of 108-132 bushels to the range of 164-201 bushels in non-irrigated land and from the range of 155-189

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[1]The expected value of irrigation water requirement is calculated by 11/2 + 14.6/5 + 16.4/10 + 18/25 + 19.1/50 = 11.16.

Table 7. Irrigation permits under Iowa water rights system, 1976[a]

	Acres irrigated	Amount granted		
		wells	reservoirs (acre feet of water)	streams
Western basin	70,651	72,522	5,422	9,462
Southern basin	17,280	12,242	125	6,942
Des Moines basin	12,944	7,257	5,163	8,490
Skunk basin	3,583	2,784	161	396
Iowa-Cedar basin	16,976	11,585	4,933	5,553
Northeast basin	2,150	1,120	587	1,438

[a]Source (5, p. 57).

Table 8. Average gross irrigation water requirements for corn for various return periods[a]

Return period years	Irrigation requirements[b] inches
2	11.0
5	14.6
10	16.4
25	18.0
50	19.1

[a]Source (64, p. 525).

[b]Assuming unlimited water after tasseling for a soil with 10.4 inches of available water in the root zone.

Table 9. Rossmiller's projected non-irrigated corn yields in Northwest Iowa for the period 1980 to 2020, bushels per acre of Class I land[a]

County	1980	2000	2020
Buena Vista	132	166	200
Cherokee	126	159	192
Clay	128	161	194
Dickinson	118	148	179
Ida	132	166	201
Lyon	108	136	164
O'Brien	125	158	191
Osceola	119	150	181
Plymouth	109	138	166
Sac	126	159	192
Sioux	114	143	173
Woodbury	116	146	176

[a]Source (64, p. 415).

Table 10. Rossmiller's projected irrigated corn yields in Northwest Iowa for the period 1980 to 2020, bushels per acre of Class I land[a]

County	1980	2000	2020
Buena Vista	189	223	258
Cherokee	182	214	248
Clay	184	217	250
Dickinson	169	199	231
Ida	189	223	258
Lyon	155	182	210
O'Brien	179	212	245
Osceola	171	202	233
Plymouth	157	185	214
Sac	182	214	248
Sioux	163	193	223
Woodbury	166	196	226

[a]Source (64, p. 415).

bushels to the range of 210-258 bushels in irrigated land. Also, irrigation is shown to boost yields a great deal; irrigation is expected to raise corn yields from the range of 164-201 bushels to the range of 210-258 bushels per acre in 2020.

Considering the wide variations of corn yields among counties, the water coefficient of irrigated corn production of a county was formed by using the following formula:

$$\text{Water coefficient} = 11.16 \times 325{,}851/(12 \times \text{yield})$$

where 325,851/12 is the conversion rate from acre-inch to gallon. The resulting water coefficients for 12 counties are reported in Table 11. If we set the corn price equal to 2.04 dollars per bushel, which was quoted for 1978, the water coefficient of corn ranges from 575.8 gallons per dollar of Buena Vista County to 707.4 of Lyon County. These figures are a great deal higher than the water requirement of 4 gallons per dollar estimated by Barnard and Dent for crop production, but substantially lower than that of crop production in California mentioned earlier. Water coefficients of corn production far exceed those of other industrial production; the water coefficient of corn production of Lyon County is nearly 50 times of the water coefficient of livestock production which is ranked top in the water coefficient list of Table 6.

In their 1976 water study, Barnard and Dent estimated the domestic ("household" by their terminology) water use rate at 53 gallons per capita per day in Iowa (5, p. 69). A rough approximation based on this

Table 11. Water and land coefficients of corn in Northwest Iowa, 2020

County	Water coeff. (gal. per bushel)	Land coefficient (acre per million bushel)	
		irrigated	non-irrigated
Buena Vista	1174.6	3876.0	5000.0
Cherokee	1221.9	4032.3	5208.3
Clay	1212.2	4600.0	5154.6
Dickinson	1311.9	4329.0	5586.6
Ida	1174.6	3876.0	4975.1
Lyon	1443.1	4761.9	6097.6
O'Brien	1236.9	4081.6	5235.6
Osceola	1300.6	4291.8	5524.9
Plymouth	1416.1	4672.9	6024.1
Sac	1221.9	4032.3	5208.6
Sioux	1358.9	4484.3	5780.3
Woodbury	1340.9	4424.8	5681.8

figure and on the average 11.16 acre-inches of annual irrigation water requirement shows that the irrigation water sprinkled over 650 acres of corn land is large enough to supply a city of 10,000 population year around in Iowa. As mentioned before, the region has only two cities with population exceeding 10,000. This tells us that a substantial increase in irrigation could impose a grueling burden on the region's water resources.

Combining the 12 water coefficients of corn production listed in Table 11 and the 12 water coefficients (excluding that of crop production) listed in Table 6 gives a total of 24 water coefficients as input into the model.

Since the reciprocal of a corn yield indicates the land requirement per unit of corn produced, it is used as the land coefficient of corn that relates corn production activities to land availability. Table 11 reports the land coefficients by types of corn. Since non-irrigated corn yields are lower than irrigated corn yields, land coefficients of non-irrigated corn are higher than those of irrigated corn.

Constraints

There are five sets of the constraints in the model: final demands, final water uses, land availability, water supplies, and minimum production requirements.

The set of final demands and final water uses are the most important part of the data series for application since they determine the level of the region's production and, thereby, determine the total water requirement of the region. The concrete contents of the region's economic activities in terms of household consumption, government activities, exports, and investment, boil down into final demands. Final uses of water constitute minimum water requirements set aside for population.

In his input-output study on the Iowa economy, Barnard estimated state final demands in 1975 that covers household consumption, government expenditures, exports, and investment (4, p. 61). From the estimates he calculated state production requirements to support the final demands. These state production requirements are reproduced in Table 12.

Such final demand data were not available for Northwest Iowa. Income data were available for this region, but in terms of 10 industrial sectors rather than 13 industrial sectors of the input-output table used in this study (see Table 3). Hence, the region's final demands were estimated from both state production data and region's income data as explained below.

The input-output assumption of a constant income coefficient (v_i) leads to proportionality between income Y_i accrued to sector i and gross production of that sector x_i, i.e.,

$$Y_i = v_i x_i \ .$$

This proportionality gives the following relation between regional production and state production:

$$x_i^r = (Y_i^r / Y_i^s) x_i^s \tag{19}$$

Table 12. Estimated gross production and final demands for the years 1975 and 2020 (in 1975 dollars)

	1975		2020 region	
	state[a]	region	gross production (x)	final demand (f)
	($ million)			
Agriculture				
livestock	3,304	717[b]	1,141	586
crop production	1,968	380[b]	604	271
Construction and mining	1,681	121	367	257
Manufacturing				
food and kindreds	4,790	326	1,044	750
other non-durables	1,976	134	431	60
farm machinery	861	59	188	171
other machinery	1,428	97	312	190
other durables	2,210	150	412	111
Transportation	847	64	189	38

[a]Source (4, p. 61).

[b]Computed from (82, p. II-13). These numbers are for 1974.

Table 12. Continued

	1975		2020 region	
	state[a]	region	gross production (x)	final demand (f)
	($ million)			
Communication and utilities	811	80	339	188
Trade	2,753	273	828	638
Finance, insurance, and real estate	2,656	215	897	543
Services	1,879	179	816	575

where

Y_i^r = income accrued to sector i of the region;

Y_i^s = income accrued to sector i of the state;

x_i^r = production of sector i of the region;

x_i^s = production of sector i of the state.

Equation (19) says that if production of a certain sector is known at state level and if income accrued to that sector are known both at the state and regional level, then production of that sector at the regional level can be estimated by scaling down production at the state level with the scaling-down factor determined by the sectoral income ratio (the ratio of regional income to state income that is accrued to the sector). The sectoral income ratio (Y_i^r/Y_i^s) is calculated sector by sector in Table 3.

Since agricultural production data of the region are available from the Census of Agriculture (82), Equation (19) was used for estimating non-agricultural sector's production of the region. For example, Table 3 shows that income from the manufacturing sector of the region is only 6.8 percent of that of the state. Since the manufacturing sector is subdivided into five subsectors in the input-output table (food and kindreds, other non-durables, farm machinery, other machinery, and other durables), each sector's production at the state level was uniformly scaled down by multiplying 6.8 percent to get an estimate of regional production of that sector in 1975. The same procedure gives estimates of other non-agricultural sector's regional production in 1975[1]. The results are reported in the second column of Table 12.

The projection of the region's production in the year 2020, reported in the second to the last column of Table 12, was formed by applying the region's sectoral income growth rates (see Table 3) to the region's 1975 production estimates[2].

Then, estimates of final demands of the region in the year 2020 were obtained, according to Equation (3), by multiplying the projected production of the region by the technical coefficient matrix of Table 5. The last column of Table 12 reports the results.

These projected final demands serve as the target level of final demand to be achieved in 2020. The final demand for food is expected to form a substantial part of the demand for manufacturing goods in the year 2020 by 750 million dollars. To satisfy this, a 3.2 times expansion of the food processing sector is required between 1975 and 2020. Agricultural products are expected to account for nearly one-fifth of the total amount of final demands which stands at 4,378 million

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[1]In income data, mining and construction are separated, while they are integrated in the input-output table. In this case, the state production was split up first according to the income share of each sector in the combined income. For example, state production of the mining sector was estimated by 1,681,389 x 50/(786+50) and that of the construction sector by 1,681,389 x 786/(786+50). Then the income ratio was applied to get estimates of regional production. The same procedure was used for the communication and utility sector.

[2]Since income coefficients are constant, an increase in income is gained through increased production.

dollars in 2020. As a result, manufacturing and agriculture are expected to still be the leading industries in the region. However, expected high demands for services (trade, finance, insurance, real estate, and other services) require a rapid expansion in these sectors, nearly a four-fold increase over the period. On the average, non-agricultural sector's expansion is noticeable, reflecting the projected vigorous growth of income generated from these sectors.

Projection of non-industrial water requirements (final uses of water) was based on the population data already presented in Table 4 and the average final water use rates presented in Table 13. Three different final water use rates were considered in terms of per capita use[1]. The urban water use rate was set 30 percent higher than the rural non-farm water use rate to reflect water losses connected with water distribution system and for other public use such as street washing, firefighting, municipal parks, swimming pools, etc. (5, p. 69)[2]. Multiplying populations of Table 4 by corresponding final water use rates of Table 13 provides projected final water use requirements of the region. Table 14 summarizes the results county by county for the

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[1]The rural water use rates in Table 13 may include a small part of non-domestic uses such as water for livestocks (64, pp. 486-488). For simplicity (and to hedge against possible higher water use rates in the future), it is assumed that the rural water use rates represent purely final uses.

[2]Adopting the same procedure, Barnard and Dent came out with 54 gallons per capita per day for the domestic ("household" in their terminology) water use rate of Iowa (5, p. 69). Including industrial and commercial uses, the urban water use rates estimated by Rossmiller are much higher than the figures for the urban water use rate as given in Table 30 (see 64, p. 488).

Table 13. Estimated final water use in Northwest Iowa

	1980	2000	2020
	(gallons per capita per day)		
Rural farm[a]	50	60	70
Rural non-farm[a]	70	80	90
Urban[b]	91	104	117

[a]Source (64, p. 488).

[b]In accordance with procedures adopted by Barnard, it is assumed that urban population use is 30 percent greater than rural non-farm water use.

years 2000 and 2020. It shows that a total of about 37 million gallons of water should be set aside daily for population of the region in 2020 prior to industrial uses. About 40 percent of this total goes to Woodbury County which has the largest population in the region.

Even though more of Iowa's farm land is expected to come under irrigation in the future, it is difficult to assess the extent and speed of expansion of irrigation due to a number of factors which resist easy prediction: future weather variability, availability of water, demands for crops, the long-term economic feasibility of irrigation, etc.

Because of the difficulty in predicting the future of irrigation in Northwest Iowa, some upper and lower limits were established on how much land will be irrigated and how much water will be applied during

Table 14. Projected final water use requirements (f_w^k) in Northwest Iowa

County	2000	2020
	(1,000 gallons per day)	
Buena Vista	2,090	2,360 (861,400)[a]
Cherokee	1,543	1,905 (695,325)
Clay	1,936	2,206 (805,190)
Dickinson	1,653	1,943 (709,195)
Ida	771	905 (330,325)
Lyon	1,271	1,737 (643,005)
O'Brien	1,802	2,135 (779,275)
Osceola	871	1,166 (425,590)
Plymouth	2,290	2,781 (1,015,065)
Sac	1,395	1,684 (614,660)
Sioux	3,189	3,873 (1,413,645)
Woodbury	12,156	14,636 (5,342,140)
Region	30,967	37,331 (13,625,815)

[a]Numbers in the parentheses are in 1,000 gallons per year. These numbers were actually used in application.

any one year[1]. Two land classes by types of land characteristics suitable for irrigation of corn were considered for irrigation, Class I and II land (64, p. 334)[2]. These two classes of land are known to be the least susceptible to erosion and the most suitable for irrigation.

As for the future irrigation, three alternative irrigation levels, referred to as irrigation level I, II, and III, were formulated. Irrigation level I assumed a ten-fold increase in irrigation between 1974 and 2020. The acreage of irrigated land of each county in 1974 is reported in Table 15. According to this, a total of 3,877 acres was irrigated in Northwest Iowa in 1974. Hence, the above assumption implies that the irrigation acreage would reach the level of 38,770 acres in 2020. Considering the rough approximation made by Hallberg, Koch, and Horick that irrigation would increase ten times in Iowa as a whole between 1976 and 2000 (29, p. 1), irrigation level I looks somewhat conservative.

Irrigation level II consists of the acreage of each county's Class I land. Table 15 shows that the acreage of Class I land of Northwest Iowa totals at 296,200 acres which is nearly 7.6 times as much as the total irrigated land under irrigation level I. In the light of

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[1] In the programming run, the minimum level was set at 5 acres. In the U.S. Census of Agriculture of 1974, 5 acres were the minimum positive level of irrigation in Northwest Iowa (82, pp. II-5-6).

[2] Class I and II land is characterized as follows (64, p. 334):
- Class I; soils with few limitations that restrict their use (with slope ranging 0.2%)
- Class II; soils with moderate limitations that reduce the choice of plants or that requires moderate conservation (with slope ranging 2-5%).

Table 15. Number of acres of irrigated crop land, Class I and II land suitable for irrigation of corn, and total crop land

County	Harvested[a] crop land irrigated (1974)	Class I[b] land (1967)	Class II[b] land (1967)	Total[b] crop land (1967)
		(acre)		
Buena Vista	135	25,400	51,300	307,437
Cherokee	0	17,500	55,400	273,717
Clay	115	40,200	14,900	300,105
Dickinson	50	19,200	21,400	178,592
Ida	0	2,200	26,100	228,198
Lyon	28	29,400	67,900	304,955
O'Brien	5	39,600	65,900	311,230
Osceola	0	29,800	38,700	217,506
Plymouth	0	11,400	58,000	431,448
Sac	158	19,200	56,000	306,360
Sioux	844	31,300	94,800	419,882
Woodbury	2,542	30,900	25,100	431,474
Region	3,877	296,200	575,500	3,709,004

[a]Source (82, p. II-3).

[b]Source (64, p. 525).

the above rough estimate by Hallberg, Koch, and Horick, irrigation level II represents a substantial increase in irrigation. Irrigation level III assumes that irrigation expands further to Class II land which totals at 575,500 acres for the region. This implies a roughly two-fold increase in irrigation over the irrigation level II.

Clay County has the largest acreage of Class I land in the region, followed by O'Brien County. Hence, these two counties have the greatest potential for irrigation in terms of land availability under the irrigation level II. Ida and Dickinson Counties have the smallest acreage of Class I and II land combined in the region.

In establishing the minimum production requirements, it was assumed that each industrial sector of each county be able to maintain at least status quo of 1975 for non-agricultural production and of 1974 for agricultural production. Some counties would have more potentials for growth, depending on water availability. As for non-agricultural production, each county's production of a certain sector in 1975 was estimated by splitting the region's total production of that sector according to that county's share of the income in the region's total income produced from the sector. The share was calculated from Table 2.

Data Set for Water Supply of Northwest Iowa

The main water supply sources considered in this chapter are grouped under two headings: surface water and ground water. One visible effect of precipitation is surface runoff, the source of most of our surface water. This water is found in rivers and streams, in natural lakes and ponds, and in man-made reservoirs. Ground water can be found near the surface in ground water table aquifers, or much deeper in confined aquifers. This vertical variation divides aquifers into two general classifications: surficial and bedrock.

Surficial aquifers can be subdivided into three main types: alluvial, buried channel and drift aquifers. Alluvial aquifers are those which lie adjacent to and beneath streams and are composed of the materials deposited by the streams. Ancient stream channels which were carved by preglacial or interglacial streams and then buried beneath the current landscape by later deposits are called buried channel aquifers. Drift aquifers are those which are located in the uplands and composed of materials deposited by glaciers. These surficial aquifers are not uniform or continuous in occurrence. They can be missing in some areas, patchy in others and thick and widespread in others.

Unlike surficial aquifers, bedrock aquifers are normally continuous and underlie large areas. They are usually composed of sedimentary rocks occurring in layers and thus areas will have two or more bedrock aquifers separated by confining layers. Portions of Iowa are underlain by three bedrock aquifers which slope from the northeast to the southwest: the Mississippian, the Silurian-Denovian and the Cambrian-Ordovician aquifers (74, pp. 29-49). Along the bottom edge of the Cambrian-Ordovician aquifers is a layer of rock known as the Jordan Sandstone which is quite productive. In Northwest Iowa, a bedrock aquifer known as the Dakota Sandstone is present and is found at a depth of only a few hundred feet.

Table 16 summarizes water supply data of the region from the preliminary report of the Iowa State Water Resource Research Institute (54). The report lists ground water availability in terms of low and upper ranges and surface water availability in terms of average annual flow and flow-duration values which are equalled or exceeded 90% and 99% of the time. This study considers only low-range ground water availability and surface water availability with 99% of the time. The report provides water supply cost data also, which are reproduced in the last row of Table 16. These costs were used as coefficients in the objective function of the model together with the income coefficients.

According to Table 16, eight of the 12 counties in the region have four water supply sources to depend on; three of these have five sources, and Woodbury County has six sources. Woodbury County has a tremendous amount of potential water supply, which totals 610,000 million gallons a year. Of this total, 500,000 million gallons can be supplied from the alluvial aquifers associated with the Missouri River flood plain at the cost of $0.75 per 1,000 gallons, which is the cheapest source in the region, because it lies near the surface and each well yields a large amount of water. In addition, this alluvial aquifer is seasonally augmented by the Missouri River which is sustained by upstream reservoirs.

Results from Application of the Model to Northwest Iowa

Table 17 summarizes production activities of the region's industrial sectors. The numbers under the heading of final demand

Table 16. Water availability of Northwest Iowa by types of supply sources and costs of water supply from each source[a]

	Bedrock GW_1	Missouri River GW_2	Surficial aquifer GW_3	Interior stream SW_1	Big Sioux River SW_3	Missouri River SW_2	Reservoir SW_4	Total
	(million gallons per year)							
Buena Vista	800	0	16,000	800	0	0	1,920	19,520
Cherokee	800	0	16,000	800	0	0	14,200	31,800
Clay	800	0	11,300	800	0	0	4,800	17,700
Dickinson	800	0	18,900	800	0	0	2,300	22,800
Ida	800	0	16,000	800	0	0	16,900	34,500
Lyon	800	0	16,000	800	600	0	13,100	31,300
O'Brien	800	0	16,500	800	0	0	9,500	27,600
Osceola	800	0	16,250	800	0	0	0	17,850
Plymouth	800	0	16,000	800	600	0	6,400	24,600
Sac	800	0	16,000	800	0	0	5,500	23,100

[a]Summarized from (54).

Table 16. Continued

	Bedrock GW_1	Missouri River GW_2	Surficial aquifer GW_3	Interior stream SW_1	Big Sioux River SW_3	Missouri River SW_2	Reservoir SW_4	Total
				(million gallons)				
Sioux	800	0	16,000	800	600	0	3,800	22,000
Woodbury	800	500,000	16,000	800	0	67,000	25,500	610,100
Water supply cost, c_i^k ($/1000 gal)	1.50	0.75	1.0	1.25	1.25	1.0	2.50	

Table 17. Final demands, gross production requirements, and income multipliers of Northwest Iowa in 2020 at 1975 prices

Industry	Final demand (f) ($ million)	Gross production (x) ($ million)	Income multiplier
Agriculture			
livestock	585.9	1,141.3	0.4529
crop production	271.2	604.3	0.3286
Construction and mining	256.9	367.0	0.7098
Manufacturing			
food and kindreds	750.2	1,044.0	0.4532
other non-durables	60.3	430.6	0.3816
farm machinery	171.4	187.5	0.5995
other machinery	190.3	312.2	0.6743
other durables	110.8	481.5	0.5831
Transportation	38.5	188.8	0.6604
Communication and Utilities	187.6	339.0	0.3674
Trade	637.6	828.1	0.6466
Finance, insurance and real estate	542.6	896.7	0.2489
Services	574.6	815.7	0.7999
Total	4,377.9	7,636.9	

indicate the levels of sectoral final demands that can be achieved subject to the region's water supply constraints. These numbers coincide with the target level of final demands of Table 12 which was imposed on the model, implying that the available water supplies of Northwest Iowa do not constitute a limiting factor to achieving the target level of final demands which was based on income growth projected for the region to the year 2020. In other words, the level of final demands projected to the year 2020 is fully feasible in terms of the available water resources the region holds.

The numbers in the second column of Table 17 indicate gross production requirements of each industrial sector to satisfy the given bill of final demands. The production requirements total at 7,636.9 million dollars. The agricultural sector accounts for 22.9% of this and the manufacturing sector for 32.2%. This projected total production requirement amounts to 28.1% of the estimate by Barnard of gross production of the whole state in 1975. The region's total income in 1975 represented only 9.2% of the state income in the same year (see Table 3).

Income multipliers are reported in the last column. The multiplier shown there indicates the increase in the total (personal) income of the region created by a one-dollar increase in final demand of the corresponding sector. Our result shows that the service sector has the highest multiplier, 0.8, while the lowest multiplier is observed in the finance and insurance sector with 0.2489. Since it is assumed that there is no income effect on production, the resulting multipliers would underestimate the actual increase in income caused by a change of final demand.

Since irrigation is by far the greatest water-consuming activity in the region, it becomes an important part of the estimation of water requirement of the region. Corresponding to three alternative irrigation levels, three programming runs were made. Table 18 shows the irrigation levels imposed on each county. The last two columns show divergences between initially imposed irrigation levels and feasible irrigation levels. Water availability prevents some counties from expanding irrigation all the way to irrigation level III which consists of the sums of the acreage of Class I and II land in each county. These include Buena Vista, Clay, O'Brien, Osceola, Sac, and Sioux Counties. As will be seen later in Table 24, an optimal water use requires these counties to stop short of exhausting all the potential water supplies and to turn to non-irrigated corn production. The maximum level of irrigation in each county feasible under each county's water availability is reported in the last column of Table 18. (The term, irrigation level III, will be retained for these maximum levels of irrigation.)

Table 19 tells us how much water would be required to support the above gross production requirements or, what is the same thing, the target level of final demands projected for the region to the year 2020. Since the interdependences among economic sectors are taken into account via the input-output system of the model, the water requirements as shown in the table include both direct and indirect requirements. Under irrigation level I, water requirements add up to 52,782 million

Table 18. Irrigation levels imposed in the model for the year 2020

	Irrigation level I	Irrigation level II	Irrigation level III feasible	(imposed)
	(acre)			
Buena Vista	1,350	25,400	58,766	(76,700)
Cherokee	50	17,500	72,900	(72,900)
Clay	1,150	40,200	53,116	(55,100)
Dickinson	500	19,300	40,700	(40,700)
Ida	50	2,200	28,300	(28,300)
Lyon	280	29,400	97,300	(97,300)
O'Brien	50	39,600	84,400	(105,500)
Osceola	50	29,800	55,222	(68,500)
Plymouth	50	11,400	69,400	(69,400)
Sac	1,580	19,200	70,220	(75,200)
Sioux	8,440	31,300	59,199	(136,100)
Woodbury	25,420	56,000	56,000	(56,000)
Region	38,970	321,300	745,523	(881,700)

Table 19. Water requirements by types of industries of Northwest Iowa in 2020

Industry	Irrigation level I	Irrigation level II	Irrigation level III
		(million gallons per year)	
Agriculture			
livestock	16,475.6 (31.2%)	16,475.6 (11.9%)	16,475.6 (6.2%)
crop production	11,851.6 (22.5%)	97,719.7 (70.5%)	226,744.3 (84.7%)
Non-agriculture	10,828.7 (20.5%)	10,828.7 (7.8%)	10,828.7 (4.0%)
construction and mining	506.6	506.6	506.6
food and kindreds	1,329.1	1,329.1	1,329.1
Other non-durables	2,005.9	2,005.9	2,005.9
farm machinery	146.4	146.4	146.4
other machinery	126.2	126.2	126.2
other durables	1,165.7	1,165.7	1,165.7
transportation	51.8	51.8	51.8
communication and utilities	4,474.3	4,474.3	4,474.3
trade	379.5	379.5	379.5
finance, insurance, and real estate	28.3	28.3	28.3

Table 19. Continued

Industry	Irrigation level I	Irrigation level II	Irrigation level III
		(million gallons per year)	
services	614.7	614.7	614.7
Final water use	13,625.8 (25.8%)	13,625.8 (9.8%)	13,625.8 (5.19%)
Total requirement	52,781.7	138,649.6	267,674.4
Ground water supplies (excluding bedrocks)	690,950	690,950	690,950
Surficial aquifers	190,950	190,950	190,950
Missouri River plain	500,000	500,000	500,000

gallons. Since irrigation level I represents a modest irrigation expansion of the region, crop production claims a modest 22.5% of the total. Due to the large livestock production requirement, the livestock sector takes up the largest portion of the total with 31.2%. The water requirements of the other industrial sectors, lumped together, represent 20.5% which is less than the amount of final water uses.

If irrigation is scaled up to level II, the total water requirement swells a great deal, to 138,650 million gallons which is a 162.7% increase over the requirement under level I. All of this increase is explained by the increase in irrigation water requirement, so that crop production accounts for a 70.5% of the total water requirement of the region. A shift from level II to level III leads to another big increase in the total water requirement solely due to increased irrigation. In irrigation level III, the total water requirement stands at 267,674 million gallons. Water for crop production claims the lion's share of the total with a 84.7% of it. This leaves only 15.3% of the total shared by all the other sectors including final uses of water.

Even though irrigation is shown to add a substantial burden to the region's water resource, it can be seen from the water supply data that the total water supply available for the region as a whole far exceeds the region's total water requirements for various production activities and final uses, even if the total requirements are scaled up by a substantial increase in irrigation and by projected population and economic growth of the region to the year 2020. Table 19 demonstrates that the amount of water that can be tapped from the alluvial aquifer of Missouri River flood plain alone, which is estimated at 500,000 million gallons per year, is a great deal more than enough to cover the region's total water requirement even under irrigation level III. Moreover, this aquifer constitutes the cheapest water supply source available for the region. Hence, it can be concluded that Northwest Iowa as a whole has potentially sufficient water supplies to depend on for the region's population and economic growth, which is not much out of the line with the projection series made by the State of Iowa.

In the subregional level, the substantial water requirement associated with expansion of irrigation may impose a heavy burden on the subregion's water resources, making water availability a limiting factor to subregional growth. Table 20 shows how the region's total production requirement presented in Table 17 is proportioned to each county. According to it, in all production categories, Woodbury County is designated as by far the largest supplier of goods and services. It takes care of nearly one-third of crop production, 37.2% of livestock production, 83.7% of non-agricultural production, and, in sum, 72.8% of the total production of the region. Livestock production and non-agricultural production by the other counties are shown to remain at the level of 1975, implying zero growth in these categories to the year 2020. Slight growth in crop production by the other counties is due to the exogenously imposed irrigation expansion. The monopoly of the region's industrial growth by Woodbury County is justified by the fact that, neighboring the Missouri River flood plain, this county has by far the largest water supply source in the region. As shown in Table 19,

Table 20. Production requirements of each county in order to achieve the level of final demands in the year 2020 under various irrigation levels

	Crop		Livestock	All other sectors	Total
	irrigation level I	irrigation level II & III			
			($ million)		
Buena Vista	50.8	50.8	41.4	118.2	210.4
Cherokee	30.0	38.0	69.3	96.4	203.7
Clay	35.7	37.5	37.5	112.2	185.4
Dickinson	22.5	22.5	24.6	88.4	135.5
Ida	27.1	27.1	41.6	45.1	113.8
Lyon	24.3	24.3	66.5	57.5	148.3
O'Brien	41.7	41.7	68.1	81.3	191.1
Osceola	22.4	25.1	40.2	40.4	103.0
Plymouth	45.3	45.3	97.1	99.2	241.6
Sac	44.0	44.0	69.4	65.6	179.0
Sioux	46.5	46.5	160.8	156.8	364.1
Woodbury	206.0	201.5	424.7	4,930.9	5,561.6
Total	604.3	604.3	1,141.2	5,892.0	7,636.7

setting aside the other water supply sources of the county, the amount of water available from the surficial aquifer of the Missouri River flood plain alone is nearly twice as much as Northwest Iowa's total water requirement under irrigation level III in 2020. Furthermore, this surficial aquifer is the cheapest water supply source in the region[1].

Table 21 describes distribution of corn production between irrigation and non-irrigation. Under irrigation level I, irrigated corn production accounts for only 6.7% of the total corn production. As irrigation is raised from level I to level II, the share of irrigated corn production sharply rises to 56.1%. Clay and Osceola Counties irrigate all of their corn production. Clay County has the largest Class I land in the region. Irrigation of all of Class I land in Osceola County exhausts the county's optimum corn production requirement which is relatively samll in the region.

Under irrigation level III, about 71.8% of the region's corn production is irrigated and most of the counties irrigate all of their corn production. Only Ida, Sac, Sioux, and Woodbury Counties retain non-irrigated land for corn production at significant levels. Ida County holds the smallest acreage of Class I and II land in the region. Sac, Sioux, and Woodbury Counties have been traditionally a large corn producer in the region.

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[1]Such large withdrawals of this surficial aquifer as indicated by Table 19 might impact eventually on the interstate allocation of the total water resource of the Missouri River basin.

Table 21. Corn production by counties in the year 2020 under various irrigation levels

	Irrigation level I		Irrigation level II		Irrigation level III	
	irrigated	non-irrigated	irrigated	non-irrigated	irrigated	non-irrigated
			(1000 bushels)			
Buena Vista	348	13,899	6,553	7,694	15,162	0
Cherokee	12	11,908	4,340	7,580	18,079	0
Clay	288	8,877	10,050	0	13,279	0
Dickinson	116	5,343	4,458	1,001	9,402	0
Ida	13	9,254	568	8,699	7,301	1,965
Lyon	59	6,765	6,174	650	20,433	0
O'Brien	12	11,406	9,702	1,716	20,678	0
Osceola	12	5,630	6,943	0	12,867	0
Plymouth	11	14,915	2,440	12,486	14,852	74
Sac	392	13,125	4,762	8,756	17,415	32,558
Sioux	1,882	14,144	6,980	9,046	13,201	2,825
Woodbury	5,745	8,368	12,656	1,457	12,656	1,457
Total	8,890	123,634	75,626	59,085	175,325	38,879

Table 22 shows that under modest irrigation level I water requirements for all types of uses are fully supplied from the cheapest water supply source available for each county. The cheapest water supply source in the region is the surficial aquifer (GW_3). Of the region's total water requirement 52,782 million gallons, 53.8% is ascribed to Woodbury County. The next largest water consumer is Sioux County which is followed by Plymouth County.

Table 23 shows water requirement of each county under irrigation level II. Woodbury County's water requirement is shown to drop to 27.2% of the region's total water requirement. This is due to large irrigation water requirement of the other counties. O'Brien, Clay, and Sioux Counties enter the list of the large water consumers in the region. The large irrigation water requirement is shown to push Clay County to the lower bound of ground and stream water supplies and forces it to turn to reservoir water which is the most expensive water supply source. Clay County holds the smallest water supply sources in the region, while it has the largest acreage of Class I land which is known to be most suitable to irrigation. All the other counties still have recourse to the cheapest water supply source (GW_3) available to each county under irrigation level II.

According to Table 24, expansion of irrigation to level III exhausts all the safe levels of ground and stream water supplies in most counties of the region (safe in the sense of the lower ranges of the ground water supply sources and the availabilities with 99% of the time from stream flows). Only Dickinson, Ida, and Osceola Counties have some

Table 22. Water requirements by counties and by types of uses under irrigation level I in 2020

County	Agricultural use crop	Agricultural use livestock	Non-agricultural use	Final water use	Total	Supply source
			(million gallons per year)			
Buena Vista	410.6	598.2	187.7	861.4	2,057.9 (3.9)[a]	GW_3
Cherokee	15.2	999.7	175.0	695.3	1,885.2 (3.6)	GW_3
Clay	349.7	541.2	199.7	805.2	1,895.8 (3.6)	GW_3
Dickinson	152.1	355.7	146.5	709.2	1,363.5 (2.6)	GW_3
Ida	15.2	600.7	73.8	330.3	1,020.0 (1.9)	GW_3
Lyon	85.2	959.8	88.4	634.0	1,767.4 (3.3)	GW_3
O'Brien	15.1	983.3	169.8	779.3	1,947.5 (3.7)	GW_3
Osceola	15.1	580.9	48.6	425.6	1,070.2 (2.1)	GW_3
Plymouth	15.2	1,401.4	176.3	1,015.1	2,608.0 (5.0)	GW_3
Sac	480.5	1,002.4	127.3	614.7	2,224.9 (4.2)	GW_3
Sioux	2,566.8	2,321.5	260.7	1,413.6	6,262.6 (11.9)	GW_3

[a]The numbers in the parentheses represent the percent of a county's total water requirement in the region's total water requirement.

Table 22. Continued

County	Agricultural use crop	livestock	Non-agricultural use	Final water use	Total T	Supply source
			(million gallons per year)			
Woodbury	7,730.0	6,130.8	9,174.9	5,342.1	28,378.7 (53.8)	GW_2
Total	11,851.6	16,475.6	10,828.7	13,625.8	52,781.7 (100.0)	

Table 23. Water requirements by counties under irrigation level II in 2020

County	Crop	Total	Supply sources
	(million gallons per year)		
Buena Vista	7,724.9	9,372.2 (6.7)	GW_3
Cherokee	5,322.1	7,192.1 (5.2)	GW_3
Clay	12,225.8	13,771.9 (9.9)	GW_1, GW_3, SW_1, SW_4
Dickinson	5,869.8	7,081.3 (5.1)	GW_3
Ida	669.1	1,673.9 (1.2)	GW_3
Lyon	8,944.9	10,627.1 (7.7)	GW_3
O'Brien	12,043.1	13,975.3 (10.1)	GW_3
Osceola	9,063.2	10,118.3 (7.3)	GW_3
Plymouth	3,467.2	6,060.0 (4.4)	GW_3
Sac	5,839.2	7,583.5 (5.5)	GW_3
Sioux	9,519.2	13,515.0 (9.8)	GW_3
Woodbury	17,031.2	37,679.0 (27.2)	GW_2
Total	98,719.7	138,469.6	

Table 24. Water requirements by counties under irrigation level III in 2020

County	Crop	Total	Supply sources
	(million gallons per year)		
Buena Vista	17,871.8	19,519.8	all
Cherokee	22,170.3	24,040.5	all
Clay	16,153.0	17,700.0	all
Dickinson	12,378.7	13,590.0	GW_3
Ida	8,606.9	9,611.7	GW_3
Lyon	29,603.3	31,285.6	all
O'Brien	25,667.6	27,000.0	all
Osceola	16,794.9	17,850.0	GW_1, GW_3, SW_1
Plymouth	21,107.1	23,700.0	all
Sac	21,355.4	23,100.0	all
Sioux	18,004.1	22,000.0	all
Woodbury	17,031.2	37,679.0	GW_2
Total	226,744.3	267,676.6	

of the water supplies left unused. Dickinson, Ida, and Osceola Counties have the smallest irrigation water requirement in the region due to the lack of land suitable for irrigation.

Table 25 shows the amount of water taken from each water supply source of the region under irrigation level II and III. Under irrigation level I, each county's total water requirement is within the supply limit of the surficial aquifer, the cheapest supply source to each county. Expansion of irrigation to level II and III forces nine counties listed in the table to reach the safe supply limit of each ground and stream water supply source and to require the most expensive water supply sources, the reservoir storage, for additional water, thereby causing the positive shadow prices to be associated with the relatively cheaper water supply sources. A shadow price of a constraint, often called the dual evaluator in the linear programming context, indicates the change in the value of the objective function (regional total income in our model) that can be achieved if the constraint were relaxed or tightened by one unit (10, p. 26). Shadow prices associated with ground and stream water under irrigation level II and III are reported in the last row of Table 25.

The surficial aquifer, which is the cheapest water supply source of each county (except for Woodbury County), has the highest shadow price, 1.5 dollars per 1000 gallons. An increase in the availability from this supply source would add to the region's total income by 1.5 dollars per 1000 gallons. Water from the stream flow, the next cheapest water supply source, has a shadow price of 1.3 dollars per 1000 gallons. In other

Table 25. Water use by types of supply sources and shadow prices of water when more than one source is tapped

	Ground water		Stream water		Reservoir	Total	
	bedrock aquifer GW_1	surficial aquifer GW_3	interior stream SW_1	Big Sioux River SW_3	SW_4	require-ment	supply
	(million gallons per year)						
Irrigation level II							
Clay	800	11,300	800	n.a.[a]	871.9	13,771.9	17,700
Irrigation level III							
Buena Vista	800	16,000	800	n.a.	1,919.8	19,519.8	19,520
Cherokee	800	16,000	800	n.a.	6,440.5	24,040.5	31,800
Clay	800	11,300	800	n.a.	4,800.0	17,700	17,700
Lyon	800	16,000	800	600	13,085.6	31,285.6	31,300
O'Brien	800	16,500	800	n.a.	9,499.9	27,599.9	27,600
Osceola	800	16,250	800	n.a.	0	17,850	17,850
Plymouth	800	16,000	800	600	5,500.0	23,700	24,600

[a]n.a. = not available.

Table 25. Continued

	Ground water		Stream water		Reservoir	Total	
	bedrock aquifer GW_1	surficial aquifer GW_3	interior stream SW_1	Big Sioux River SW_3	SW_4	requirement	supply
	(million gallons per year)						
Sac	800	16,000	800	n.a.	5,500.0	23,100	23,100
Sioux	800	16,000	800	600	3,800.0	22,000	22,000
Shadow price ($/1000 gal)	1.00	1.50	1.30	1.30	0		

words, the water from the stream flow is worth 1.3 dollars per 1000 gallons to the region's economy. The water from bedrock aquifers, the most expensive one besides the water from reservoirs, is worth one dollar per 1000 gallons. No county exhausts the full reservoir capacity, so that the water from reservoirs has a zero shadow price. An additional reservoir capacity over the level specified in Table 16 makes no contribution to the region's income generation and thus, is unnecessary except for the extreme drought period (i.e., 99 percent of the time).

One important result is that the water from alluvial aquifers of the Missouri River flood plain, the cheapest water supply source in the region as a whole, still commands a zero shadow price, even if it is put under the region-wide water distribution. Since irrigation constitutes the main burden on the water supplies of most counties in the region and since, as a result, most counties turn to relatively expensive water supply sources to meet increased water requirement, it is desirable to impose as much irrigation water requirement as possible on the surficial aquifer of the Missouri River flood plain. This implies that Woodbury County, on the Missouri River flood plain, should expand irrigation as much as possible. However, availability of land suitable for irrigation in Woodbury County limits its expansion of irrigation. Table 18 shows that this county has only 6.3 percent of the region's total acreage of Class I and II land combined.

Another possibility is to transport the water from the surficial aquifers of the Missouri River flood plain to other counties. These other counties have recourse to the water supply sources which are more costly (by more than 33 percent) than the water from the Missouri River flood plain.

Table 16 shows that the water supply cost from these surficial aquifers is estimated at 0.75 dollars per 1000 gallons and that from interior streams at 1.25 dollars per 1000 gallons. Hence, if the transport cost is less than the difference between these two costs (0.5 dollars per 1000 gallons), transportation of the water from the Missouri flood plain would save the costs associated with utilizing relatively expensive water supply sources by the other counties including interior streams, bedrock aquifers, Big Sioux River, and reservoir storages. If the transport cost is less than 1.75 dollars per 1000 gallons, which is the difference between the above 0.75 dollars and the cost of the water from the reservoir storage, transporting the water from the surficial aquifers of the Missouri River flood plain dispenses with building expensive reservoir facilities in the other counties.

CHAPTER IV. EXTENSION OF THE MODEL FOR IMPACT AND MULTI-OBJECTIVE ANALYSIS

The Extended Input-output System

Extension of the basic input-output system

The input-output system is based on inter-sectoral transactions. The principal features of the flow of inter-sectoral transactions are shown in Table 26 (81, p. 24). The items in the second and third quadrant of the table are defined as follows:

C_i = household consumption expenditures on final output produced by sector i;

G_i = government expenditures on final output produced by sector i;

I_i = investment by sector i;

E_i = exports by sector i;

Y_i = factor payment by sector i;

T_i = taxes paid by sector i;

S_i = saving and depreciation allowances in sector i; and

M_i = imports by sector i.

The items in the fourth quadrant are explained in the footnote[1].

---

[1] Y_C = intrahousehold transactions; S_C = household savings and expenditures for depreciation of consumer durables or capital goods; M_C = household purchase of imports; Y_G = factor payment by government (e.g., wages and salaries of government employees and interest payment by governments) and various transfer payments by governments; T_G = tax payment by governments (e.g., employment taxes); S_G = depreciation allowances for public facilities and government surplus; T_I = sales and excise taxes on purchases of new capital goods and consumer durables; S_I = net investment (see 3, pp. 79-80); M_I = imported capital goods and net lending to foreign countries; Y_E = net flow of factor income from foreign countries; S_E = capital exports; M_E = balance of trade.

Table 26. Generalized transaction table

	Producing sector	Final demand (expenditure for)	Total
Producing sector	(I) x_{11} .... x_{1n} . . . . . . x_{n1} x_{nn}	(II) C_1 G_1 I_1 E_1 . . . . C_n G_n I_n E_n	x_1 . . . x_n
Payment to	(III) Y_1 ..... Y_n T_1 ..... T_n S_1 ..... S_n M_1 ..... M_n	(IV) Y_C Y_G Y_E T_C T_G T_I S_C S_G S_I S_E M_C M_G M_I M_E	Y T S M
Total	x_1 ..... x_n	C G I E	X

Summing across the totals row and down the totals column,

$$X = \sum_{j}^{n} x_j + C + G + I + E,$$

and

$$X = \sum_{i}^{n} x_i + Y + T + S + M.$$

Since $\sum_j x_j = \sum_i x_i$ and all intermediate flow totals are canceled out,

$$Y + T + S + M = C + G + I + E.$$

That is, value-added + imports = final demand.

Consider again an economy with m different primary and natural resources and with n industrial (producing) sectors, each producing one homogeneous commodity. The following notation will be repeatedly used:

$x = (x_1, x_2, \ldots, x_n)'$ = vector of gross outputs of the producing sectors;

Y = personal disposable income;

$f = (f_1, f_2, \ldots, f_n)'$ = vector of final outputs of the producing sectors = vector of autonomous spending on final outputs (= commodity expenditures, briefly);

$r = (r_1, r_2, \ldots, r_m)'$ = vector of total use of primary and natural resources (measured in physical units);

$f_r = (f_{r1}, f_{r2}, \ldots, f_{rm})'$ = vector of final use of primary and natural resources (measured in physical units);

G = autonomous non-commodity expenditures (G will be defined later);

$v = (v_1, v_2, \ldots, v_n)'$ = vector of factor costs per unit of gross output = vector of income coefficient of x;

$c = (c_1, c_2, \ldots, c_n)'$ = vector of the disaggregated marginal propensity to consume final outputs;

$c_r = (c_{r1}, c_{r2}, \ldots, c_{rm})'$ = vector of the disaggregated marginal propensity to consume primary and natural resources;

$\hat{c}$ = the aggregated marginal propensity to consume the primary and natural resources altogether;

$X' = (x', r', Y)$;

$d' = (f', f_r', G)$;

$\hat{d}' = (f', f_r', \hat{n}G)$, where $\hat{n} = 1/(1-\hat{c})$;

$\hat{v} = \hat{n}v$;

$A = ((a_{ij}))$ = nxn technical coefficient matrix;

$B = ((b_{ij}))$ = mxn matrix of primary and natural resource requirement per unit of gross outputs;

$u' = v'(I-A)^{-1}$ = vector of income multipliers for f; and

$z = (I-A)^{-1}c$ = vector of induced production.

It is not necessary, but convenient, to assume that r and f_r are measured in physical units like gallons, tons, etc. Autonomous spending includes household autonomous consumption expenditures, government spending, investment, and exports. Autonomous consumption expenditures are grouped into two categories: those on produced commodities and those on primary and natural resources (including environmental resources). As income rises, people would spend more not only on produced commodities, but also on natural resources.

Since primary and natural resources are the economy's only income earning inputs and all income is due to the sale of them, Y consists of payments made by all the industrial sectors and non-industrial sectors for primary and natural resources. The first row of the quadrant III and IV of Table 26 suggests the following income equation:

$$Y = Y_1 + Y_2 + \ldots\ldots Y_n + Y_C + Y_G + Y_E$$

where

Y_i = income received from producing (industrial) sector i, $i = 1, 2, \ldots, n$;

Y_C = income received from the household sector;

Y_G = income received from governments (e.g., wages and salaries of governments, various transfer payments by governments, etc.); and

Y_E = income received from the foreign sector.

Income received from the household sector (Y_c) is identically equal to what the household sector pays primary and natural resources for their services. Assume that this household expenditure is related to income Y as in the following expression:

$$Y_c = \hat{c}Y + \overline{C} \ ,$$

where $\overline{C}$ represents an autonomous component. Substituting this into the above income equation,

$$Y = Y_1 + Y_2 + \ldots + Y_n + \hat{c}Y + G$$

where

$$G = \overline{C} + Y_G + Y_E \quad .$$

We will simply call G autonomous non-commodity expenditures. Putting $\hat{n} = 1/(1 - \hat{c})$ revises the above equation as

$$Y = \hat{n}(Y_1 + Y_2 + \ldots + Y_n + G)$$

Let x_{ij} and r_{ij} denote the amount of x_i and r_i to be used for production in sector j, respectively. By the bookkeeping identity of the input-output table,

$$x_i = x_{i1} + x_{i2} + \ldots + x_{in} + c_iY + f_i \;(i = 1, 2, \ldots, n) \qquad (20a)$$

$$r_i = r_{i1} + r_{i2} + \ldots + r_{in} + c_{ri}Y + f_{ri} \;(i = 1, 2, .., m) \qquad (20b)$$

$$Y = \hat{n}(Y_1 + Y_2 + \ldots + Y_n) + \hat{n}G \qquad (20c)$$

Equations (20a) and (20b) say that gross output of sector i and resource i are allocated for intermediate uses (x_{ij} and r_{ij}), consumptive uses (c_iY and $c_{ri}Y$) induced by changes in income, and final uses associated with autonomous spending.

Now, let's introduce the proportionality assumption:

$$x_{ij} = a_{ij}x_j$$
$$r_{ij} = b_{ij}x_j$$
$$Y_j = v_jx_j$$

where a_{ij}, b_{ij}, and v_j are constants. This assumption allows the equation system (20) to be rewritten as follows[1]:

$$
\begin{array}{llllll}
(1-a_{11})x_1 & -a_{12}x_2 \cdots\cdots & -a_{1n}x_n & & & -c_1Y = f_1 \\
-a_{21}x_1 & +(1-a_{22})x_2 \cdots\cdots & -a_{2n}x_n & & & -c_2Y = f_2 \\
\vdots & \vdots & \vdots & & & \vdots \quad \vdots \\
-a_{n1}x_1 & -a_{n2}x_2 & +(1-a_{nn})x_n & & & -c_nY = f_n \\
-b_{11}x_1 & -b_{12}x_2 & -b_{1n}x_n & +r_1 & & -c_{r1}Y = f_{r1} \\
\vdots & \vdots & \vdots & & \ddots & \vdots \quad \vdots \\
-b_{m1}x_1 & -b_{m2}x_2 & -b_{mn}x_n & & +r_m & -c_{rm}Y = f_{rm} \\
-\hat{n}v_1x_1 & -\hat{n}v_2x_2 & -\hat{n}v_nx_n & & & +Y = \hat{n}G
\end{array}
$$

or in matrix notation

$$(I - D)X = \hat{d} \tag{21}$$

where

---

[1]If we assume linearity instead of proportionality as

$$x_{ij} = a_{ij}x_j + \bar{x}_{ij},\ r_{ij} = b_{ij}x_j + \bar{r}_{ij},$$

$\bar{x}_{ij}$ and $\bar{r}_{ij}$ are included in f_i and f_{ri}, respectively.

$$X = \begin{pmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_n \\ r_1 \\ \cdot \\ \cdot \\ \cdot \\ r_m \\ Y \end{pmatrix}, \quad \hat{d} = \begin{pmatrix} f_1 \\ \cdot \\ \cdot \\ \cdot \\ f_n \\ f_{r1} \\ \cdot \\ \cdot \\ \cdot \\ f_{rm} \\ \hat{n}G \end{pmatrix}, \quad D = \begin{pmatrix} a_{11} & \cdots & a_{1n} & 0 & 0 & \cdots & c_1 \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} & 0 & 0 & \cdots & c_n \\ b_{11} & \cdots & b_{1n} & 0 & 0 & \cdots & c_{r1} \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ b_{m1} & \cdots & b_{mn} & 0 & 0 & \cdots & c_{rm} \\ \hat{v}_1 & \cdots & \hat{v}_n & 0 & 0 & \cdots & 0 \end{pmatrix}$$

and $\hat{v}_i = \hat{n}\, v_i$.

The (n+m+1)x(n+m+1) matrix (I - D) will be referred to as the augmented Leontief matrix.

If the bundle of final demand $\hat{d}$ is exogenously specified, Equation (21) yields

$$X = (I - D)^{-1}\hat{d} \qquad (22)$$

if the inverse exists. This inverse will be referred to as the augmented Leontief inverse. The inverse translates the given bundle of final demands into the equilibrium amounts of economy's gross production (x), employment of non-produced resources (r), and total income (Y).

When an input-output table is made for a particular year, positive x, f, Y, and G are actually observed for that year. Let non-negative D be computed for a particular year and let non-negative X and $\hat{d}$ be observed for the year as well. Then condition (1) of Theorem 1 is satisfied for (I - D). It follows that there exists an inverse $(I - D)^{-1}$ so that for any final demand vector $\hat{d}$ Equation (22) holds.

Since the first n columns and rows of (I - D) form the Leontief matrix (I - A), (I - D) is partitioned as

$$\left(\begin{array}{c|c} I-A & -C_o \\ \hline -\bar{R} & E \end{array}\right)$$

where

$$\bar{R} = \begin{pmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & & \vdots \\ b_{m1} & \cdots & b_{mn} \\ \hat{v}_1 & \cdots & \hat{v}_n \end{pmatrix}, \quad C_o = \begin{pmatrix} 0 & 0 & & 0 & c_1 \\ 0 & 0 & & 0 & c_2 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & c_n \end{pmatrix}, \quad E = \begin{pmatrix} 1 & 0 & \cdots & 0 & -c_{r1} \\ 0 & 1 & \cdots & 0 & -c_{r2} \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -c_{rm} \\ 0 & 0 & \cdots & 0 & 1 \end{pmatrix}$$

There exists E^{-1};

$$E^{-1} = \begin{pmatrix} 1 & 0 & \cdots & 0 & c_{r1} \\ 0 & 1 & \cdots & 0 & c_{r2} \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & c_{rm} \\ 0 & 0 & \cdots & 0 & 1 \end{pmatrix}$$

Let $(I - D)^{-1}$ be partitioned in the same way as (I - D):

$$(I - D)^{-1} = \left(\begin{array}{c|c} T & U \\ \hline S & W \end{array}\right)$$

By definition of the inverse matrix,

$$\begin{bmatrix} (I - A) & -C_o \\ -\bar{R} & E \end{bmatrix} \begin{bmatrix} T & U \\ S & W \end{bmatrix} = \begin{bmatrix} I_n & 0 \\ 0 & I_{m+1} \end{bmatrix}$$

Four equations are obtained for the four unknown submatrices T, U, S, and W (28, pp. 108-109):

$$(I-A)T - C_oS = I_n$$

$$(I-A)U - C_oW = 0$$

$$-\bar{R}T + ES = 0$$

$$-\bar{R}U + EW = I_{m+1}$$

Since there exists E^{-1}, the third equation yields

$$S = E^{-1}\bar{R}T$$

Substituting this into the first equation,

$$(I - A - C_oE^{-1}\bar{R})T = I_n$$

By definition of the inverse matrix,

$$T = (I - A - C_oE^{-1}\bar{R})^{-1}$$

From the existence of $(I - D)^{-1}$ and E^{-1} follows existence of $(I - A - C_oE^{-1}\bar{R})^{-1}$. Following the same procedures, we obtain

$$T = (I - A - C_oE^{-1}\bar{R})^{-1}$$

$$U = (I - A - C_oE^{-1}\bar{R})^{-1}C_oE^{-1}$$

$$S = E^{-1}\bar{R}(I - A - C_oE^{-1}\bar{R})^{-1}$$
$$W = E^{-1} + E^{-1}\bar{R}(I - A - C_oE^{-1}\bar{R})C_oE^{-1} .$$

Due to the special pattern of E^{-1}, $C_oE^{-1} = C_o$ and $C_oE^{-1}\bar{R} = C_o\bar{R} = c\hat{v}'$.

Hence,

$$(I - D)^{-1} = \left[\begin{array}{c|c} (I-A-c\hat{v}')^{-1} & (I-A-c\hat{v}')^{-1}C_o \\ \hline E^{-1}\bar{R}(I-A-c\hat{v}')^{-1} & E^{-1} + E^{-1}\bar{R}(I-A-c\hat{v}')^{-1}C_o \end{array}\right]$$

Since

$$E^{-1}\bar{R} = E^{-1}\left[\begin{array}{c} B \\ \hline \hat{v}' \end{array}\right] = \left[\begin{array}{c} B+c_r\hat{v}' \\ \hline \hat{v}' \end{array}\right]$$

we have

$$(I-D)^{-1} = \left[\begin{array}{c|c|c} (I-A-c\hat{v}')^{-1} & 0 & (I-A-c\hat{v}')^{-1}c \\ \hline (B+c_r\hat{v}')\ (I-A-c\hat{v}')^{-1} & I_m & c_r+(B+c_r\hat{v}')\ (I-A-c\hat{v}')^{-1}c \\ \hline \hat{v}'(I-A-c\hat{v}')^{-1} & 0 & 1+\hat{v}'(I-A-c\hat{v}')^{-1}c \end{array}\right] \quad (23)$$

where 0 is a null matrix or a null vector.

The solutions of the open system

Suppose that $c = 0$ and $\hat{c} = 0$, i.e., a change in income does not create a change in consumption of commodities and of non-produced resources[1]. Let's call the resulting extended input-output system the open system. Accordingly, $\hat{n} = 1$, and $v = \hat{v}$. This assumption greatly simplifies (I-D) and $(I-D)^{-1}$ as follows:

[1] As a result, all the induced consumption is transferred into the final demand category.

$$(I-D) = \left[\begin{array}{c|c} (I-A) & 0 \\ \hline -B & \\ \cline{1-1} -v' & I_{m+1} \end{array}\right] \tag{24}$$

$$(I-D)^{-1} = \left[\begin{array}{c|c|c} (I-A)^{-1} & 0 & 0 \\ \hline B(I-A)^{-1} & I_m & 0 \\ \hline v'(I-A)^{-1} & 0 & 1 \end{array}\right] \tag{25}$$

In the extended open system, the (nxn) submatrix in the upper left cell of the augmented inverse matrix is the Leontief inverse of the basic open model. Plugging the above inverse matrix into Equation (22) yields solutions for gross production (x), resource employment (r), and income (Y) for given values of final demands f, f_r, and G:

$$x = (I - A)^{-1}f \tag{26}$$

$$r = B(I - A)^{-1}f + f_r = Bx + f_r \tag{27}$$

$$Y = v'(I - A)^{-1}f + G = v'x + G \tag{28}$$

Equation (26) is identical to Equation (4) of the basic input-output system. Furthermore, Equations (27) and (28) show that, once x is determined by Equation (26), r and Y are also determined. That is, the solution of the basic input-output system is a basic solution from which solutions for additional two endogenous variables (r and Y) of the open system are derived. In particular, if the Leontief inverse of the basic input-output system, $(I-A)^{-1}$, is known, all the solutions of the open system are directly obtained from the basic input-output system.

Equation (27) is related to the concept of the production possibility curve (transformation curve) in the input-output model. It says that if f and f_r are given, r is determined. On the other hand, if r is fixed, the problem is to choose a set of f and f_r such that all the fixed resources will be fully employed. The set of f and f_r is constrained by the availability of primary resources r. Therefore, the set of solutions for f and f_r satisfying

$$B(I-A)^{-1}f + f_r \leq r$$

with r fixed provides the production - possibility set associated with particular value of r. For instance, in a two-sector and two resource system where

$$B(I-A)^{-1} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}, \quad f = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}, \quad r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix},$$

and $f_r = 0$, solutions for f_1 and f_2 satisfying

$$h_{11}f_1 + h_{12}f_2 \leq r_1$$
$$h_{21}f_1 + h_{22}f_2 \leq r_2$$

describes the production-possibility set OABC in Figure 2. Point B is the only one where both r_1 and r_2 are fully employed. Along the frontier AB, r_2 is fully employed while some of r_1 is unemployed, and along the frontier BC, vice versa.

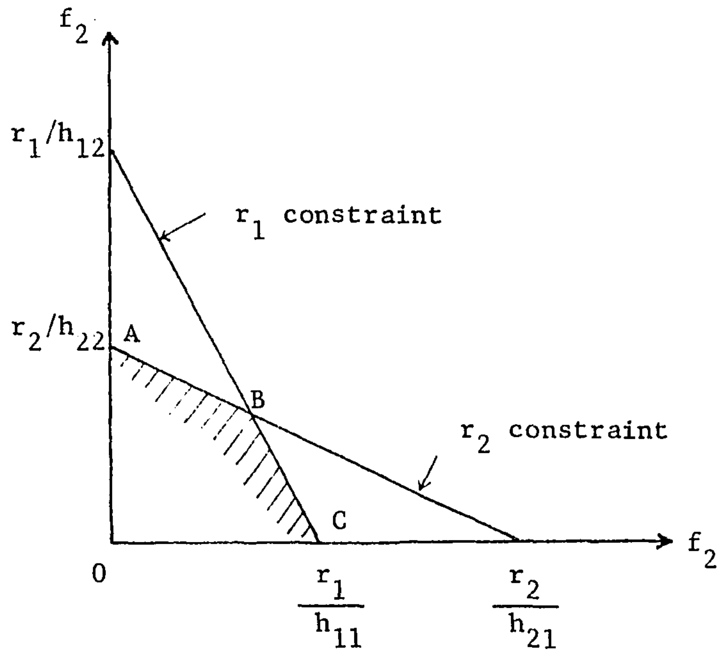

Figure 2. The production possibility set

While A, B, and v relate gross output x to production, resource employment, and income, respectively, $(I-A)^{-1}$, $B(I-a)^{-1}$, and $v'(I-A)^{-1}$ relate final outputs f to production, resource employment, and income, respectively. An increase in f_i requires not only an increase in x_i directly, but also increases in x_j for j = 1, 2, ..., n indirectly due to interdependence in production. The latter set of matrices reflect both the direct and the indirect requirements associated with f. Since an increase in expenditures on f (commodity expenditures) has a multiplier effect on production, resource employment, and income to the extent of $(I-A)^{-1}$, $B(I-A)^{-1}$, and $v'(I-A)^{-1}$, respectively, these matrices can be alternatively referred to as the production multiplier matrix, the resource-employment multiplier matrix, and the income multiplier vector for f, respectively.

Equations (26) - (28) show that the autonomous non-commodity expenditure G does not have a multiplier effect in the open system. Even if the government boosts household income through subsidies and transfer payments or through other measures, it does not affect the economy's production, hence, employment. This is because a mechanism to link income to production is wiped out by the assumption of c = o and $\hat{c}$ = o. The only way for the government to change the level of the economy's production and employment is by its purchases of produced commodities. The open system explains how effects of such purchases propagate to production and employment.

<u>The solutions of the closed system</u>

Assume that $c \neq 0$ and $\hat{c} \neq 0$. Then, a change in income would entail a change in consumption which in turn touches off a series of repercussions on production, employment, income, and consumption again. Taking these repercussions into consideration leads to what will be called the closed system. For ready reference, the inverse of Equation (23) is reproduced here:

$$(I-D)^{-1} = \left[\begin{array}{c|c|c} (I-A-c\hat{v}')^{-1} & 0 & (I-A-c\hat{v}')^{-1}c \\ \hline (B+c_r\hat{v}')(I-A-c\hat{v}')^{-1} & I_m & c_r+(B+c_r\hat{v}')(I-A-c\hat{v}')^{-1}c \\ \hline \hat{v}'(I-A-c\hat{v}')^{-1} & 0 & 1+\hat{v}'(I-A-c\hat{v}')^{-1}c \end{array}\right]$$

where $\hat{v} = \hat{n}v$.

A brief comparison of this $(I-D)^{-1}$ with $(I-D)^{-1}$ of the open system would yield an observation that introduction of income effect transforms $(I-A)^{-1}$, A, and B of the open system into $(I-A-c\hat{v}')^{-1}$, $(A+c\hat{v}')$, and $(B+c_r\hat{v}')$, respectively.

Since

$$c\hat{v}' = ((c_i\hat{v}_j)) \text{ and } c_r\hat{v}' = ((c_{ri}\hat{v}_j)),$$

the i-j th element of $(A+c\hat{v}')$ and $(B+c_r\hat{v}')$ turns out to be $(a_{ij}+c_i\hat{v}_j)$ and $(b_{ij}+c_{ri}\hat{v}_j)$, respectively, indicating that each unit of product j is associated with $(a_{ij}+c_i\hat{v}_j)$ units of sector i's product and $(b_{ij}+c_{ri}\hat{v}_j)$ units of resource i rather than only with a_{ij} and b_{ij} units as in the open system.

Substituting the inverse of Equation (23) into (22) provides the solutions of the extended closed system for x, r, and Y. However, such a solution process leads to the following troubles in input-output analysis. First, it involves the procedures of computing the inverse matrix, thus adding to the computational burden. This burden would become heavy in terms of time and money especially when the size of the matrix is huge. In order to reduce the computation time and cost, the method of matrix inversion by partitioning can be used. However, the augmented Leontief inverse of Equation (23) comprises another inverse matrix $(I-A-c\hat{v}')^{-1}$ in itself. Computing the inverse calls for computing another inverse.

Second, even if the inverse matrix had been computed, its usefulness is limited, because the vector of the disaggregated marginal propensity to consume (MPC) is congealed into the resulting inverse matrix. The MPC vector is considered to be more variable than technical coefficients (84, p. 43). This makes the inverse matrix also variable. This in turn subjects the resulting input-output solutions to the same variability[1]. Everytime the MPC vector is revised, the inverse should be computed anew. Therefore, it is desirable to separate the term $c\hat{v}'$ from $(I-A-c\hat{v}')^{-1}$. Another advantage of separating the MPC vector from $(I-A-c\hat{v}')^{-1}$ is that Leontief inverse of the basic input-output system, $(I-A)^{-1}$, is provided by any input-output table, so that the augmented Leontief, $(I-D)^{-1}$, can be spelled out without following tedious and costly procedures of matrix inversion.

To separate $c\hat{v}'$ from $(I-A-c\hat{v}')^{-1}$, the following useful theorem can be employed (26, p. 211).

Theorem 2: If A is a kxk non-singular matrix and c and d are kx1 vectors, then $|A+cd'| = |A|(1+d'A^{-1}c)$. If the inverse of the matrix $(A+cd')$ exists, the inverse is given by

$$(A+cd')^{-1} = A^{-1} - \frac{(A^{-1}c)(d'A^{-1})}{1 + d'A^{-1}c}$$

Applying this theorem to $(I-A-c\hat{v}')$ gives

$$(I-A-c\hat{v}')^{-1} = (I-A)^{-1} + \frac{[(I-A)^{-1}c][\hat{v}'(I-A)^{-1}]}{1 - \hat{v}'(I-A)^{-1}c} \qquad (29)$$

It is simple to show that the denominator is positive. By Theorem 2,

[1]Miyazawa deals with this problem in length (52).

$$|I-A-c\hat{v}'| = |I-A|\ [I-\hat{v}(I-A)^{-1}c]\ .$$

By the Hawkins-Simon condition (see Theorem 1 of p. 16), all the principal minors of (I-A) and (I-D) are positive so that

$$|I-A-c\hat{v}'| > 0 \quad \text{and} \quad |I-A| > 0\ .$$

Therefore, by Theorem 2,

$$1 - \hat{v}'(I-A)^{-1}c > 0\ . \qquad \text{QED.}$$

Since $(I-A)^{-1} \geq 0$ by Theorem 1 and both c and $\hat{v}$ are non-negative, the following inequality can be proved as a by-product:

$$(I-A-c\hat{v}')^{-1} \geq (I-A)^{-1}\ .$$

That is, the effect of autonomous spending on production, employment, and income is greater in the closed system than in the open system.

Substituting Equation (29) into Equation (13), we obtain after a little manipulation (see Appendix)

$$(I-D)^{-1} = \left[\begin{array}{c:c:c} (I-A)^{-1} + \dfrac{(I-A)^{-1}c\hat{v}'(I-A)^{-1}}{1 - \hat{v}'(I-A)^{-1}c} & 0 & \dfrac{(I-A)^{-1}c}{1 - \hat{v}'(I-A)^{-1}c} \\ \hdashline B(I-A)^{-1} + \dfrac{[B(I-A)^{-1}c+c_r]\hat{v}'(I-A)^{-1}}{1 - \hat{v}'(I-A)^{-1}c} & I_m & \dfrac{B(I-A)^{-1}c+c_r}{1 - \hat{v}'(I-A)^{-1}c} \\ \hdashline \dfrac{\hat{v}'(I-A)^{-1}}{1 - \hat{v}'(I-A)^{-1}c} & 0 & \dfrac{1}{1 - \hat{v}'(I-A)^{-1}c} \end{array}\right] \tag{30}$$

Thus, the augmented inverse matrix is stated in terms of already known coefficients, $(I-A)^{-1}$, B, c, c_r, and $\hat{v}$. The vector of marginal propensity to consume is separated from matrix inversion. Hence, revision of c does not entail computation of a new inverse. In fact, there is no need to compute any inverse matrix in obtaining the augmented Leontief inverse matrix because $(I-A)^{-1}$ is readily available from any input-output table.

Separating $\hat{n}$ from $\hat{d} = (x', r', \hat{n}G)'$ and putting it into $(I-D)^{-1}$ revise the last column of $(I-D)^{-1}$ as

$$\begin{pmatrix} \dfrac{(I-A)^{-1}c\hat{n}}{1 - \hat{v}'(I-A)^{-1}c} \\ \dfrac{[B(I-A)^{-1}c+c_r]\hat{n}}{1 - \hat{v}'(I-A)^{-1}c} \\ \dfrac{\hat{n}}{1 - \hat{v}'(I-A)^{-1}c} \end{pmatrix}$$

Since $\hat{n} = 1/(1-\hat{c})$ and $\hat{v} = \hat{n}v = v/(1-\hat{c})$,

$$\frac{\hat{n}}{1 - \hat{v}'(I-A)^{-1}c} = \frac{1/(1-\hat{c})}{1 - v'(I-A)^{-1}c/(1-\hat{c})} = \frac{1}{1 - [\hat{c}+v'(I-A)^{-1}c]}$$

Similarly,

$$\frac{\hat{v}'(I-A)^{-1}}{1 - \hat{v}'(I-A)^{-1}c} = \frac{v'(I-A)^{-1}/(1-\hat{c})}{1 - v'(I-A)^{-1}c/(1-\hat{c})} = \frac{v'(I-A)^{-1}}{1 - [\hat{c}+v'(I-A)^{-1}c]}$$

Plugging these results back into Equation (30),

$$(I-D)^{-1} = \left[\begin{array}{c|c|c} (I-A)^{-1} + Mzu' & 0 & Mz \\ \hline B(I-A)^{-1} + M(Bz+c_r)u' & I_m & M(Bz+c_r) \\ \hline Mu' & 0 & M \end{array}\right] \qquad (31)$$

where

$$u' = v'(I-A)^{-1},$$

$$z = (I-A)^{-1}c \ ,$$

and

$$M = \frac{1}{1 - [v'(I-A)^{-1}c + \hat{c}]} = \frac{1}{1 - (u'c+\hat{c})} \qquad (32)$$

Notice that the vector of final demands associated with this new inverse matrix is $d' = (x', r', G)$. Substituting the inverse in Equation (31) into Equation (22) gives the following closed system solutions:

$$x = [(I-A)^{-1} + Mzu']f + (Mz)G \qquad (33)$$

$$r = [B(I-A)^{-1} + M(Bz+c_r)u']f + f_r + M(Bz+c_r)G \qquad (34)$$

$$Y = (Mu')f + MG \qquad (35)$$

If $c=0$ and $\hat{c}=0$, then $M=1$ and the above solutions reduce to

$$x = (I-A)^{-1}f$$

$$r = B(I-A)^{-1}f + f_r$$

$$Y = u'f + G$$

which coincide with the solution formulas of the open system in Equations (26) - (28) for exogenously specified final demands f, f_r, and G.

The column sum of $(I-A)^{-1}$ is called the type I output multiplier. That is, if $(I-A)^{-1}$ is denoted by E, the sum of the elements of $E_{.j}$, where $E_{.j}$ is the j-th column of E, becomes the type I output multiplier of sector j. Since our model has two different types of autonomous consumption spending, f and G, there are two different types of the type II output multiplier; one for f and another for G. The type II output multiplier for f is the column sum of $[(I-A)^{-1}+Mzu']$.

Suppose that f and G increase by Δf and ΔG, respectively. Let's denote the resulting increments in x, r, and Y by Δx^c, Δr^c, and ΔY^c, respectively, when $c \neq 0$ and $\hat{c} \neq 0$, and by Δx^o, Δr^o, and ΔY^o, respectively, when $c = 0$ and $\hat{c} = 0$. Then,

$$\Delta x^c - \Delta x^o = Mz(u'\Delta f+\Delta G) \tag{33b}$$

$$\Delta r^c - \Delta r^o = M(Bz+c_r)(u'\Delta f+\Delta G) \tag{34b}$$

$$\Delta Y^c - \Delta Y^o = (M-1)(u'\Delta f+\Delta G) \tag{35b}$$

If $c = 0$ and $\hat{c} = 0$, of course, the right-hand sides vanish and $\Delta x^c = \Delta x^o$, $\Delta r^c = \Delta r^o$, and $\Delta Y^c = \Delta Y^o$. Hence, the expressions on the right-hand side of Equations (33b) - (35b) reflect the income effects, indicating additional production and resource requirements and the resulting increase in income created purely by the income effect. The difference between the type II and I output multipliers is measured by

Mz of Equation (33b). M appears in every expression of the right-hand side.

<u>The multiplier process of the closed system vs. that of the open system</u>

In Equation (35)

$$Y = Mu'f + MG \ ,$$

M is instantly identified as a multiplier equivalent to the Keynesian multiplier. The equation says that, for example, if the government increases its direct income payment to households by ΔG, a multiplier process would increase income by $M\Delta G$ ultimately. Similarly, if the government increases its expenditures on produced commodities by Δf, the equation predicts an ultimate increase in income by $Mu'\Delta f$. Depending on the nature of expenditure, two kinds of multipliers are recognized; M is the multiplier for non-commodity expenditures (G) and Mu' is the multiplier for commodity expenditures (f).

Figure 3 is presented to assist tracing out the multiplier process underlying the multipliers.

Suppose that commodity expenditures f be increased by Δf. Production (x) needs to be expanded to meet the increase in the autonomous demand. So the initial injection Δf is directed to x in the loop diagram of Figure 3. Production increases by Δf. This requires $A\Delta f$ to be produced as intermediate inputs. (The technical coefficient matrix A represents intermediate input requirements.) But, to produce $A\Delta f$ requires $A^2\Delta f$ as intermediate inputs, and to produce $A^2\Delta f$ requires $A^3\Delta f$ as inputs and so on. So, total production to satisfy the initial increase in f is

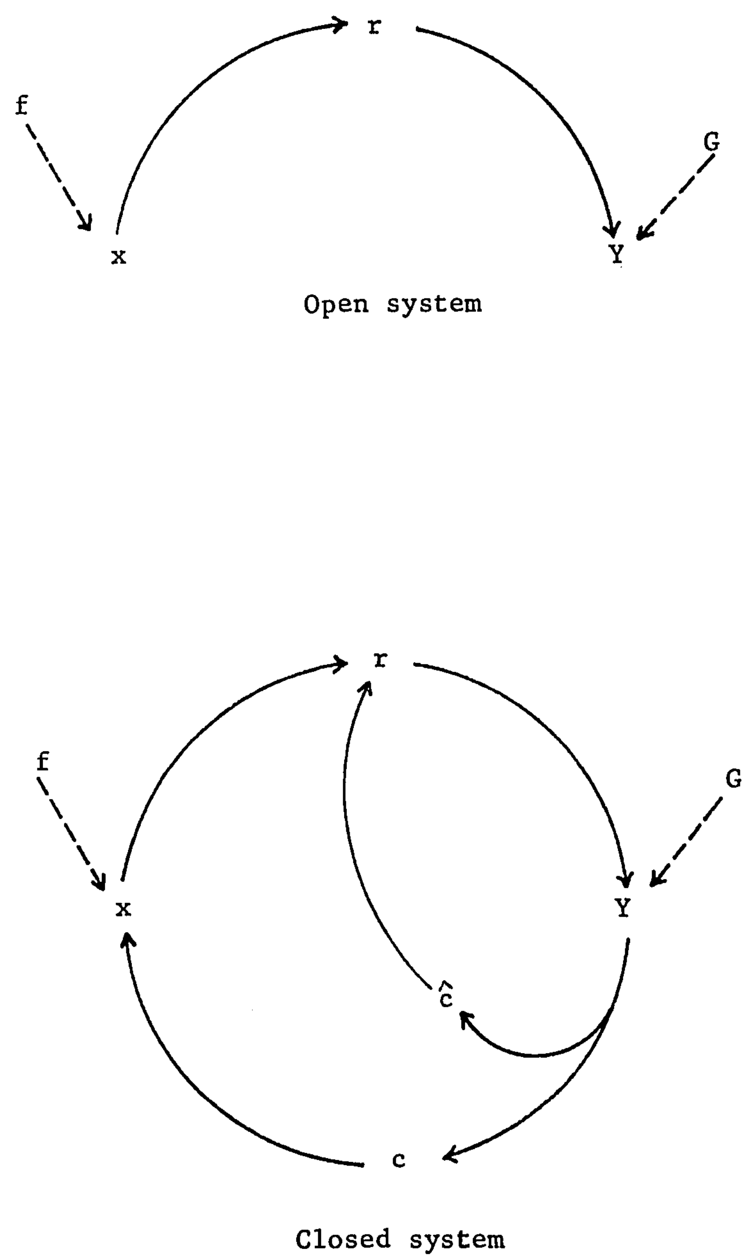

Figure 3. Circular flow in the economy underlying the open input-output system and the closed input-output system

$$(I + A + A^2 + \ldots\ldots\ldots)\Delta f = (I-A)^{-1}\Delta f \tag{36}$$

if such an inverse exists. However, production requires not only intermediate inputs, but also primary and natural resources. Hence, the effect of production expansion propagates to resource employment (r) and then to income (Y) along the upper half loop of Figure 3. Since B and v are the resource requirement matrix and the income coefficient vector (of x), respectively, increases in resource employment and in income would be

$$B(I + A + A^2 + \ldots\ldots)\Delta f = B(I - A)^{-1}\Delta f \ , \tag{37}$$

and

$$v'(I + A + A^2 + \ldots\ldots)\Delta f = v'(I - A)^{-1}\Delta f = u'\Delta f \ , \tag{38}$$

respectively. Thus, an increase in commodity expenditures by Δf ends up in an increase in income by $u'\Delta f$ along the upper half loop. This makes the vector u a multiplier relevant to the upper half loop. If $c = 0$ and $\hat{c} = 0$, the propagation process stops here, because there is no mechanism to connect the increase in income at the end of the upper half loop to another round of consumption expenditures along the lower half loop and then back to the upper half loop. As is shown in the open system loop in Figure 3, Y is not linked to x. An increase in non-commodity expenditures (G) increases income dollar by dollar, but does not affect the level of production because income is not linked to production. There is no term for G in Equation (26). Non-commodity expenditures do not have any multiplier effect in the open system.

The results in Equations (36) - (38) are exactly the open system solutions obtained when the open system Equations (26) - (28) are solved for $f = \Delta f$. In fact, these results are what multiplier analysis of the open system is about. It is restricted to the upper half loop only.

If $c \neq 0$ and $\hat{c} \neq 0$, then the increase in income at the end of the upper half loop (the first round increase in income) propagates further down along the lower half loop and the small inside loop in Figure 3. On the one hand, the increase in income, which is equal to $u'\Delta f$, gives rise to consumption expenditures of size $cu'\Delta f$ on commodities. These induced consumption expenditures become a new bill for production, opening propagation along the upper half loop at x. This propagation along the upper half loop starts with $cu'\Delta f$ and proceeds as spelled out in Equations (36) - (38) with Δf replaced by $cu'\Delta f$.

On the other hand, the first round increase in income ($u'\Delta f$) also gives rise to consumption expenditures of size $\hat{c}u'\Delta f$ on resources. Since these expenditures are what resources receive as income, they result in an equivalent increase in income. So, the process along this line of expenditures forms a small inside loop in Figure 3.

Therefore, the second round increases in income associated with the initial injection of Δf add up to

$$u'cu'\Delta f + \hat{c}u'\Delta f = (u'c + \hat{c})u'\Delta f$$

which constitutes an injection into the third round. Consumption expenditures on commodities would rise by $c(u'c + \hat{c})u'\Delta f$ and those on resources by $\hat{c}(u'c + \hat{c})u'\Delta f$. Production and employment should expand to meet those induced commodity expenditures, generating an increase in income by $u'c(u'c + \hat{c})u'\Delta f$. Since $\hat{c}(u'c + \hat{c})u'\Delta f$ directly increases income by an equal amount, the third round increases in income add up to

$$u'c(u'c + \hat{c})u'\Delta f + \hat{c}(u'c + \hat{c})u'\Delta f = (u'c + \hat{c})^2 u'\Delta f$$

which again becomes an injection into the fourth round. Writing out the successive rounds of income increases, we have

$$\Delta Y = [1 + (u'c+\hat{c}) + (u'c+\hat{c})^2 + \ldots.]u'\Delta f \ . \tag{39}$$

Since $0 \leq (u'c+\hat{c}) < 1$ (by Theorem 2), the series converges and, therefore,

$$\Delta Y = \frac{u'}{1-(u'c+\hat{c})} \Delta f \quad \text{or} \quad Y = Mu'\Delta f \ , \tag{40}$$

which is exactly what Equation (35) gives for $f = \Delta f$ and $G = 0$.

Suppose that non-commodity expenditures G rather than commodity expenditures f be increased initially by ΔG. This increase may be in the form of direct income payments by the government to households or in the form of increased resource uses. In any case, income rises by ΔG. Therefore, an initial injection of ΔG is directed to Y in the loop diagram of Figure 3 and follows the same propagation process along the whole loop as was traced out for Δf. The only difference is that the first round propagation process starts along the lower half loop with an initial increase in income of size ΔG rather than $u'\Delta f$ as in the case of an autonomous increase in commodity expenditure f. Hence, Equations (39) and (40) are replaced by

$$\Delta Y = [1 + (u'c+\hat{c}) + (u'c+\hat{c})^2 + \ldots.] \, \Delta G \tag{39b}$$

and

$$\Delta Y = \frac{1}{1-(u'c+\hat{c})} \Delta G \quad \text{or} \quad \Delta Y = M\Delta G \, . \tag{40b}$$

Again, this result is exactly what Equation (35) gives for $G = \Delta G$ and $f = 0$.

The propagation process of autonomous spending forms a 'closed loop' in the closed system, while it forms an 'open loop' in the open system. Two types of the multiplier process are unveiled accordingly; along the upper half loop, the multiplier process involving production and, along the lower half loop, the one involving consumption expenditures. The closed system covers both types. Since the open system covers only the former type along the upper half loop, it gives only one (income) multiplier, u, which explains by how much income multiplies as a result of production. Let's call this (vector) multiplier 'the sub-multiplier', because it accounts for only what takes place within the producing sectors when autonomous spending is changed.

Dividing the open system (income) multiplier for f, u, by income coefficients of x gives the type I income multipliers as follows:

$$u'D_v^{-1} = v'(I-A)^{-1}D_v^{-1}$$

where D_v is a diagonal matrix whose i-th diagonal element is v_i (income coefficient of x_i). Similarly, since the closed system (income) multiplier for f is Mu', the so-called type II income multipliers are given as

$$Mu'D_v^{-1} .$$

Obviously, once the type I income multipliers are known, the type II income multipliers are obtained simply by multiplying the type I by M. Hirsh first pointed out that the type II is a constant multiple of the type I (36). Bradley and Gander proved this rigorously (8). Our result shows that the constant is none other than M which is equivalent to the Keynesian multiplier of the orthodox Keynesian macro-model, as will be shown below.

<u>The input-output multiplier process vs. the Keynesian multiplier process</u>

Let's call M the input-output Keynesian multiplier. To examine the relationship between M and the orthodox Keynesian multiplier denoted by K, assume that $u_i = 1$ for all i, so that u' = (1, 1, ..., 1). Then,

$$M = \frac{1}{1-(u'c+\hat{c})} = \frac{1}{1 - c^*} = K$$

where c* is the aggregate marginal propensity to consume. Thus, the input-output Keynesian multiplier becomes the Keynesian multiplier of the orthodox Keynesian macro-model. This is true when a leak in the expenditure stream is introduced. For instance, when there exist some imports, K turns out to be

$$K = \frac{1}{1 - (c^*-m^*)}$$

where m* is the marginal propensity to import and c* now stands for the aggregated marginal propensity to consume domestic goods.

In our input-output system, Equation (20a) is now replaced by

$$(I-A)x - (c-m)Y = f$$

and the multiplier for G becomes

$$M = \frac{1}{1-[u'(c-m)+\hat{c}]} , \qquad (41)$$

where c and m are vectors of the disaggregated marginal propensity to consume domestic goods and to import, respectively. Again, if $u_i = 1$ for all i, then

$$M = \frac{1}{1-(c^*-m^*)} = K . \qquad (42)$$

Hence, if elements of the sub-multiplier u (open system income multipliers for f) are all unity so that an increase in commodity expenditures results in an increase in income by the same amount (along the upper half loop), then M coincides with the orthodox Keynesian multiplier. In fact, in this case, no difference is made in multiplier effects by how the bundle of f is composed of what kinds of commodities. For example, if the government increases its purchase of commodity i and j by one dollar divided in the proportion of (1-p) to p between the two, then

$$(1-p)u_i + pu_j = (1-p) + p = 1$$

whatever p is. Furthermore, no difference is made by whether autonomous spending is on f or G. Since

$$u'f = \sum_{i}^{n} f_i ,$$

u'f is the total commodity expenditures. Equation (35) now becomes

$$Y = Mu'f + MG = M(\sum_{i}^{n} f_i + G) = M \bar{G} = K \bar{G} ,$$

where $\bar{G}$ is the total autonomous expenditures without a particular reference to whether the expenditures are in the form of G or f. The multiplier process of the closed input-output system coincides with that of the Keynesian model.

In general, however, different industries have not only different income coefficients (v_i's), but also different production coefficients (e_{ij}'s) of final outputs, so that u_i may not be uniform for all i, much less unity for all industries. In fact, the strength and purpose of input-output analysis is to capture these kinds of individual characteristics for each producing sector. The sub-multiplier u quantifies them in the input-output multiplier process. Therefore, a difference is made in multiplier effects not only by whether an autonomous expenditure is in the form of f or G, but also by how the bundle of f is composed of what kinds of commodities.

Multiplier analysis discussed so far assumes no leakage in the sub-multiplier process (along the upper half loop). This may not be the case, for example, if a part of the intermediate inputs is imported.

In the input-output system, let F_i be the amount of imported intermediate input i. Then, Equation (20a) is rewritten as[1]

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[1] See Chenery and Clark (12, pp. 23-25).

$$x_i + F_i - \Sigma a_{ij}x_j - c_iY = f_i \quad (i=1, 2, \ldots, n) \tag{20b}$$

Assume that

$$F_i = \bar{F}_i + m_ix_i$$

where m_i is the import coefficient. Then Equation (20b) becomes

$$(I+F-A)x - cY = f ,$$

where f includes $\bar{F}_i$, and F is a diagonal matrix whose i-th diagonal element is m_i. As a result, the Leontief inverse $(I-A)^{-1}$ is replaced throughout all previous equations by $(I+F-A)^{-1}$, so that the sub-multiplier u is redefined as

$$u' = v'(I+F-A)^{-1} .$$

In the input-output system, the leakage due to import of intermediate inputs is absorbed into the Leontief inverse matrix and, hence, the sub-multiplier u.

In the Keynesian framework, Miyazawa's treatment of the imported intermediate inputs is briefed here (52). He defined

R = total intermediate inputs used;

$\bar{a}$ = total intermediate input requirement per dollar of total gross outputs; and

t = the self-sufficing ratio of intermediate inputs (i.e., the ratio of the amount of home-supplied inputs to R).

Since he did not deal with the marginal propensity to consume resources, let $\hat{c} = 0$. Suppose that autonomous demand increase by $\Delta\bar{G}$ regardless of whether it is in a form of f or G[1]. Initially, production should expand by $\Delta\bar{G}$ to meet the demand, which requires $t\bar{a}\Delta\bar{G}$ as home-supplied intermediate input. But, to produce $t\bar{a}\Delta\bar{G}$ requires $(t\bar{a})^2\Delta\bar{G}$ as inputs, and to produce $(t\bar{a})^2\Delta\bar{G}$ requires $(t\bar{a})^3\Delta\bar{G}$ and so on. Hence, total production to satisfy the initial increase in autonomous demand is

$$[1 + t\bar{a} + (t\bar{a})^2 + (t\bar{a})^3 + \ldots]\,\Delta\bar{G} = \frac{1}{1 - t\bar{a}}\,\Delta\bar{G}\,. \qquad (43)$$

Since $(1-\bar{a})$ is the value-added ratio (i.e., the amount of income generated per dollar of total gross outputs), total increase in income associated with the production is

$$(1-\bar{a})\,[1 + (t\bar{a}) + (t\bar{a})^2 + \ldots.]\,\Delta\bar{G} = \frac{1 - \bar{a}}{1 - t\bar{a}}\,\Delta\bar{G}\,. \qquad (44)$$

This equation is a generalization of Equation (38). The ratio, $(1-\bar{a})/(1-t\bar{a})$, is the Keynesian counterpart of the sub-multiplier u. Notice that

$$(1-\bar{a})/(1-t\bar{a}) \leq 1$$

where equality holds when t = 1, i.e., when there is no leakage in the sub-multiplier process. Miyazawa formulated the Keynesian multiplier process as follows:

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[1]Miyazawa did not differentiate f and G.

$$\Delta Y = [\bar{u} + \bar{u}^2(c^*-m^*) + \bar{u}^3(c^*-m^*)^2 + \ldots.]\Delta\bar{G} = \frac{\bar{u}}{1-\bar{u}(c^*-m^*)}\Delta\bar{G} \qquad (45)$$

where

$$\bar{u} = (1-\bar{a})/(1-t\bar{a}) \ .$$

Our multiplier analysis discussed so far shows that the multiplier process Miyazawa depicted is relevant only to the autonomous demand for commodities, i.e., relevant only when $\Delta\bar{G} = \Delta f$. Figure 4 explains this. Since the first term of the series in Equation (45) (the initial increase in income) is $\bar{u}\,\Delta\bar{G}$, by implication, the starting point of the multiplier process was set at x in the loop diagram.

Suppose that non-commodity expenditure be increased by ΔG (i.e., $\Delta\bar{G} = \Delta G$). Since this increase is simultaneously what resources receive as income, the initial increase in income is ΔG. The starting point of the multiplier process is Y in the loop diagram. The corresponding multiplier process is demonstrated by

$$\Delta Y = [1 + \bar{u}(c^*-m^*) + \bar{u}^2(c^*-m^*)^2 + \ldots.]\ \Delta G$$

$$= \frac{1}{1-\bar{u}(c^*-m^*)}\Delta G \ . \qquad (46)$$

If it is assumed that $\bar{u} = 1$, the series in both Equations (45) and (46) reduce to the following familiar Keynesian multiplier formula:

$$1 + (c^*-m^*) + (c^*-m^*)^2 + \ldots. = \frac{1}{1-(c^*-m^*)} \ .$$

The sub-multiplier process recedes out of sight because the Keynesian sub-multiplier is unity by assumption and only the multiplier process

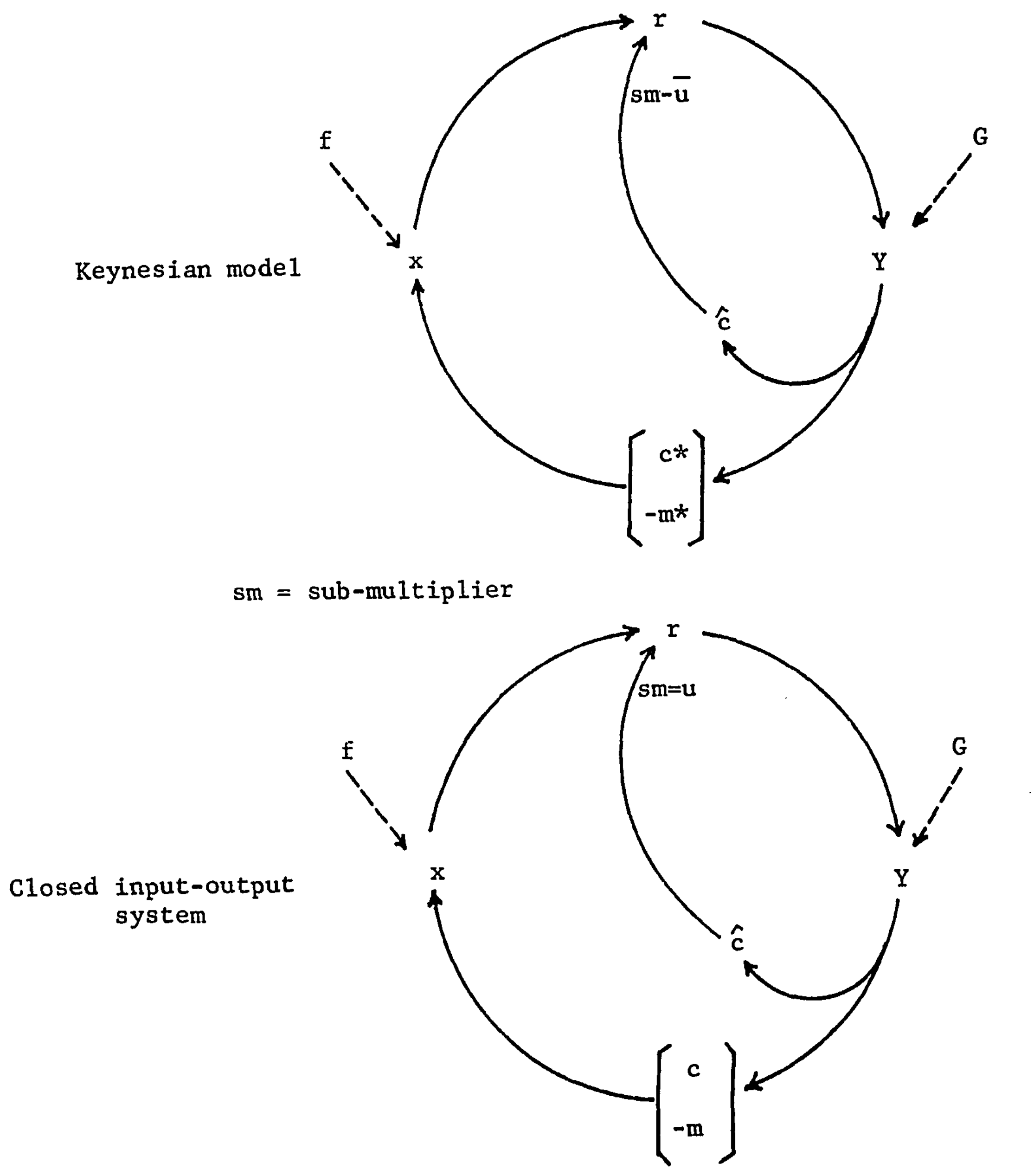

Figure 4. Circular flow in the economy underlying the orthodox Keynesian macro-model and the (closed) input-output system

involving consumption spending along the lower half loop is made explicit.

Suppose that the marginal propensity to consume resources is not zero ($\hat{c} \neq 0$). Then, tracing out the multiplier process along the loop of Figure 4 revises Equations (45) and (46) as follows:

$$\Delta Y = \{1 + [\bar{u}(c^*-m^*)+\hat{c}] + [\bar{u}(c^*-m^*)+\hat{c}]^2 + \ldots\} \bar{u}\Delta f$$

$$= \frac{\bar{u}}{1-[\bar{u}(c^*-m^*)+\hat{c}]} \Delta f \tag{45b}$$

and

$$\Delta Y = \{1 + [\bar{u}(c^*-m^*)+\hat{c}] + [\bar{u}(c^*-m^*)+\hat{c}]^2 + \ldots\} \Delta G$$

$$= \frac{1}{1-[\bar{u}(c^*-m^*)+\hat{c}]} \Delta G \quad . \tag{46b}$$

When f and G simultaneously increase by Δf and ΔG, respectively, we have

$$\Delta Y = K\bar{u}\Delta f + K\Delta G$$

for the Keynesian macro-model, and

$$Y = Mu'\Delta f + M\Delta G$$

for the closed (input-output) system, where

$$K = \frac{1}{1-[\bar{u}(c^*-m^*)+\hat{c}]} \quad \text{and} \quad M = \frac{1}{1-[u'(c-m)+\hat{c}]} \quad .$$

Now, it is clear that there exists a perfect correspondence in style between the Keynesian multipliers ($K\bar{u}$ and K) of the orthodox Keynesian macro-model and the input-output Keynesian multipliers (Mu' and M). The only difference between them is that $\bar{u}$, c*, and m* are scalars, while u, c, and m are vectors. This is natural in that the orthodox Keynesian model is an aggregated model, while the input-output model is a disaggregated model. If each u_i (the sub-multiplier of sector i) of u is assumed equal to $\bar{u}$ (the sub-multiplier of the Keynesian model), i.e., if it is assumed that a one-dollar increase in commodity expenditures, regardless of whether they are on commodity i or j, ends up uniformly in a $\bar{u}$ dollar increase in income in the sub-multiplier process, then the input-output Keynesian multiplier M becomes identical to the orthodox Keynesian multiplier K and the closed input-output system collapses into the Keynesian model. There still remains, however, a difference between the multiplier effect of commodity expenditures (f) and that of non-commodity expenditures. That is, a different composition of the expenditure would lead to a different multiplier effect on the economy. If $\bar{u}$ and u_i are further assumed equal to unity for all i, then this difference vanishes, as shown before.

Impact Analysis and Multi-objective Analysis

Granted that there exists a shortage of a resource and that competition exists for the use of existing supplies of the resource, one of the problems is to achieve efficiency in allocation of the resource. If efficiency is attained, a maximum social value of goods and services will flow from a given quantity of the resource. Two types of analytical methods on resource allocation can be juxtaposed: marginal analysis and linear programming.

Standard economic theory characterized by marginal analysis asserts that, for an efficient allocation, the resource should be so allocated that all uses derive equal value in use from the marginal unit used. In the regime visualized by this marginal analysis, production factors are continuously substitutable for each other. Thus, if the amount of one factor employed be reduced by a small amount, it will be possible to maintain the quantity of output by a small increase in the amount of the other factors employed. Moreover, each successive unit decrement in the amount of a factor will require a slightly larger increment in the amount of the factor that is substituted, if output is to remain constant.

A different production regime is visualized by linear programming (and input-output analysis). In this regime, the quantity of output is in fixed relations with the quantities of factor inputs. Factors cannot be substituted for each other except by changing the levels of various outputs, because each production uses inputs in fixed ratios. Usually, linear programming does not seek to determine directly the optimal quantity of each factor input, but, instead, the optimal levels of each production. From these levels the inputs quantities follow in due course. Accordingly, output substitution plays a role analogous to that of factor substitution in marginal analysis (16, pp. 143-144). In fact, linear programming frequently uses the notion of a 'process' or 'activity', the notion of a specific method for performing an economic task. The essential simplification achieved in input-output analysis and linear programming is the replacement of the mysterious notion of production function in marginal analysis by the hard-headed notion of the process or activity. The process is an observable unit of activity and can be empirically estimated without elaborate analysis. Linear programming and input-output analysis belong in the application-oriented economic theory.

Linear programming model incorporating income effect

A direct substitution of the augmented Leontief inverse of Equation (23) into Equation (22) provides the following alternative expressions of the closed system solutions:

$$x = (I-A-c\hat{v}')^{-1}(f+c\hat{n}G) \qquad (33c)$$

$$r = (B+c_r\hat{v}')(I-A-c\hat{v}')^{-1}(f+c\hat{n}G) + f_r + c_r\hat{n}G \qquad (34c)$$

$$Y = \hat{v}'(I-A-c\hat{v}')^{-1}(f+c\hat{n}G) + \hat{n}G \qquad (35c)$$

These solutions can be also expressed as follows:

$$\begin{pmatrix} x \\ r \\ Y \end{pmatrix} = (I-D_c)^{-1} \begin{pmatrix} f \\ f_r \\ \hat{n}G \end{pmatrix} + (I-D_c)^{-1} \begin{pmatrix} c \\ c_r \\ 0 \end{pmatrix} \hat{n}G$$

$$= (I-D_c)^{-1} \begin{pmatrix} f+\hat{n}G \\ f_r+c_r\hat{n}G \\ \hat{n}G \end{pmatrix} ,$$

where

$$(I-D_c)^{-1} = \left[\begin{array}{c|c|c} (I-A-c\hat{v}')^{-1} & 0 & 0 \\ \hline (B+c_r\hat{v}')(I-A-c\hat{v}')^{-1} & I_m & 0 \\ \hline \hat{v}'(I-A-c\hat{v}')^{-1} & 0 & 1 \end{array}\right] .$$

Since

$$(I - D_c) = \left[\begin{array}{c|c} (I-A-c\hat{v}') & 0 \\ \hline -(B+c_r\hat{v}') & \\ \cline{1-1} -\hat{v}' & I_{m+1} \end{array}\right] ,$$

Equations (33c) - (35c) can be rewritten as

$$(I-A-c\hat{v}')x = f + c\hat{n}G \tag{33d}$$

$$-(B+c_r\hat{v}')x + r = f_r + c_r\hat{n}G \tag{34d}$$

$$-\hat{v}'x + Y = \hat{n}G \quad . \tag{35d}$$

Suppose that the supplies of resources are given by $\bar{r}$. Uses of resources must be limited by this resource availability:

$$r \leq \bar{r} .$$

Hence, imposing the resource supply constraints revises Equation (34d) as follows:

$$(B+c_r\hat{v}')x \leq \bar{r}-(f_r+c_r\hat{n}G) \quad . \tag{34e}$$

Making Equation (35d) the objective function and replacing Equation (34d) by (34e) leads to the following linear programming problem that incorporates the income effect into the linear programming model applied in the previous chapter:

$$\text{Max } Y = \hat{v}'x + \hat{n}G$$

subject to

$$(I-A-c\hat{v}')x + s_x = f + c\hat{n}G$$

$$(B+c_r\hat{v}')x + s = \bar{r} - (f_r+c_r\hat{n}G) \tag{47}$$

$$x \geq 0,\ s_x \geq 0,\ s \geq 0$$

or

$$\left[\begin{array}{c|c|c} (I-A-c\hat{v}') & 0 & I_n \\ \hline (B+c_r\hat{v}') & I_m & 0 \end{array}\right] \begin{bmatrix} x \\ s \\ s_x \end{bmatrix} = \begin{bmatrix} f+c\hat{n}G \\ \bar{r}-(f_r+c_r\hat{n}G) \end{bmatrix} \tag{47b}$$

$$x \geq 0,\ s_x \geq 0,\ s \geq 0\ ,$$

where $s_x' = (s_{x1}, s_{x2}, \ldots, s_{xn})'$ and $s' = (s_1, s_2, \ldots, s_m)'$ are vectors of slack variables. The vector s_x, if it is positive, represents by how much production falls short of final demand. Solving the linear programming problem (47) provides solutions for gross production and resource employment with the income effect incorporated, and also a set of shadow prices associated with all the resource constraints of the model.

Linear programming is applied extensively to real-world problems, not because the real world is linear, but because the technique is very powerful. A major aspect of the method's power is the information about the impact of a change in the parameters on the solutions. For instance, suppose that a change takes place in supply of a certain resource. Due to interdependences among economic sectors, such a change can have pervasive effects on the economy's production, overall resource employment, and, hence, income. Since the objective function is addressed in terms of income maximization, the impact of the change on the level of income is related to the idea of the shadow price, which is a useful concept for imputing a price to the resource constraint. If decisions on resource allocation are complicated by considerations of other objectives or non-economic constraints (for example, legal, physical, and political constraints) in addition to the efficiency objective, it would be of interest to see how such multiple objectives affect allocation of a particular resource and the resulting shadow price. In this case, an impact analysis would allow us to figure out trade-off's among objectives.

It is possible to consider a variety of variations in the parameters (for example, variations in the coefficients of the objective function, in the technical coefficients, in the resource requirement coefficients, in resource availability, etc.) and conduct corresponding impact analysis (14, 70). The impact analysis presented below is on the case where there arises a shortage of a certain resource (due to increased demand or decreased supply or both) and where considerations of additional objectives other than the stated efficiency objective are involved.

Linear programming model for impact analysis

Suppose that the economy's current production is in equilibrium. Production is in equilibrium if it is just equal to the quantity demanded for all purposes: consumption, investment, inventories, exports, and so on. As shown in the basic input-output system, such equilibrium is characterized by the input-output solution, regardless of a reference to a particular objective or objectives. In terms of the linear programming problem (47), the equilibrium is described by $s_x = 0$. Such de facto equilibrium in current production must have been feasible with respect to the current resource supplies, so that $s \geq 0$ (i.e., $r \leq \bar{r}$). Then, from Equation (47b)

$$\begin{bmatrix} x \\ s \end{bmatrix} = Q^{-1} \begin{bmatrix} f + c\hat{n}G \\ \bar{r} - (f_r + c_r\hat{n}G) \end{bmatrix}, \tag{48}$$

where

$$Q = \left[\begin{array}{c|c} (I - A - c\hat{v}') & 0 \\ \hline (B + c_r\hat{v}') & I_m \end{array}\right].$$

Since

$$Q^{-1} = \left[\begin{array}{c|c} (I - A - c\hat{v}')^{-1} & 0 \\ \hline -(B + c_r\hat{v}')(I - A - c\hat{v}')^{-1} & I_m \end{array}\right],$$

the economy's production and resource employment are given by

$$x = (I - A - c\hat{v}')^{-1}(f + c\hat{n}G) \qquad (49)$$

$$s = \bar{r} - [(B + c_r\hat{v}')(I - A - c\hat{v}')^{-1} + f_r + c_r\hat{n}G] \ . \qquad (50)$$

Equation (49) is the same as the Equation (33c). Notice that the bracketed term in Equation (50) represents the total resource requirements r under the input-output solution, so that

$$s = \bar{r} - r \geq 0 \ .$$

Now, suppose that due to an increase in resource demand or a decrease in resource supply or both, a shortage surges up in resource j (like the recent energy crunch). Such a shortage of a particular resource calls for an overall reallocation of each resource because of interdependences among resource uses. However, as pointed out before, in the face of the resource shortage, linear programming does not seek to determine directly the optimal reallocation of each resource input, but, instead, the optimal re-adjustment of each production in the light of an objective or objectives. That is, the adjustment to the resource shortage is carried out through "output substitution". Therefore, the relevant question is how to trim off each production in order to meet the new resource supply and demand situation.

To answer this question, the following spade work is in order. Instead of specifying n slack variables, s_x, in the linear programming problem (47), let's introduce a dummy sector which controls the economy's overall production adjustment to the resource shortage. Let x_{id} be an "initial" reduction of production by sector i and interpret x_{id} as the amount purchased by the dummy sector from sector i. Let x_d be the initial reduction of the economy's total production necessitated by the resource shortage. Then, x_d represents the total amount of purchase by the dummy sector. By how much the economy's total production would be "ultimately" reduced depends on the size of the multipliers of the economy. Each sector's initial production reduction should add up to the total initial production reduction of the economy. Hence,

$$x_d = x_{id} + x_{2d} + \dots + x_{nd} .$$

Further, let

$$x_{id} = \hat{a}_i x_d .$$

Then,

$$\hat{a}_1 + \hat{a}_2 + \dots + \hat{a}_n = 1 .$$

The (nx1) vector $\hat{a}$ whose i-th component is $\hat{a}_i$ represents the proportion in which the initial reduction of total production spreads among sectors. For example, $\hat{a} = (\frac{1}{2}, \frac{1}{2}, 0, \dots, 0)'$ indicates that the initial reduction of the economy's total production is evenly divided between sector 1 and sector 2. What the value of individual $\hat{a}_i$ would be is crucial, because it determines the direction and magnitude of the impacts of the stated resource shortage. This will be fully discussed in the next subsection. For the time being, vector $\hat{a}$ is regarded as the result of purposeful allocation decisions under the possible multiple objectives and thus is taken as exogenously specified. One of the purposes of this subsection is to show how a given $\hat{a}$ affects the economy's production, resource employment, and income.

To summarize the spade work so far in the linear programming jargon, the slack variable s_j of s exits out of basis because the j-th resource constraint is binding, and the variable x_d replaces s_x to enter the basis.

Therefore, our new linear programming problem becomes

$$\max \quad Y = \hat{v}x' + \hat{n}G$$

subject to

$$Q_c \begin{bmatrix} x \\ \hat{s} \end{bmatrix} = \begin{bmatrix} f + c\hat{n}G \\ \bar{r} - (f_r + c_r\hat{n}G) \end{bmatrix} \qquad (51)$$

$$x \geq 0, \ \hat{s} \geq 0 \ ,$$

where Q_c is a (n+m)x(n+m) matrix such that

$$Q_c = \left[\begin{array}{c|ccccccc} (I - A - c\hat{v}') & 0 & 0 & \dots & \hat{a}_1 & \dots & 0 \\ & \cdot & \cdot & & & & \cdot \\ & \cdot & \cdot & & & & \cdot \\ & \cdot & \cdot & & & & \cdot \\ & 0 & 0 & \dots & \hat{a}_n & \dots & 0 \\ \hline (B + c_r\hat{v}') & 1 & 0 & \dots & 0 & \dots & 0 \\ & 0 & 1 & \dots & 0 & \dots & 0 \\ & \cdot & \cdot & & \cdot & & \cdot \\ & \cdot & \cdot & & \cdot & & \cdot \\ & \cdot & \cdot & & \cdot & & \cdot \\ & 0 & 0 & \dots & 0 & \dots & 1 \end{array}\right]$$

and

$$\hat{s} = (s_1, s_2, \ldots, s_{j-1}, x_d, s_{j+1}, \ldots, s_m)' .$$

The (nx1) sub-vector of the (n+j)th column of Q_c is vector $\hat{a}$ and all other elements of it are zero. The solution that maximizes Y is given by

$$\begin{bmatrix} x \\ \hat{s} \end{bmatrix} = Q_c^{-1} \begin{bmatrix} f + c\hat{n}G \\ \bar{r} - (f_r + c_r\hat{n}G) \end{bmatrix} \tag{52}$$

The matrix Q_c can be rewritten as

$$Q_c = Q + ae'_{n+j}$$

where

$$a = \begin{bmatrix} \hat{a}_1 \\ \vdots \\ \hat{a}_n \\ 0 \\ \vdots \\ 0 \\ -1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} n+j \qquad e_{n+j} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} n+j$$

By Theorem 2 again,

$$Q_c^{-1} = Q^{-1} - \frac{Q^{-1}ae'_{n+j}Q^{-1}}{1 + e'_{n+j}Q^{-1}a} = Q^{-1} + \frac{1}{k_j}[Q^{-1}ae'_{n+j}Q^{-1}] \tag{53}$$

where

$$k_j = -(1 + e'_{n+j}Q^{-1}a)$$

Equation (52) is rewritten as

$$\begin{bmatrix} x \\ \hat{s} \end{bmatrix} = Q_c^{-1} \begin{bmatrix} f + c\hat{n}G \\ \bar{r} - (f_r + c_r\hat{n}G) \end{bmatrix} \tag{54}$$

$$= Q^{-1} \begin{bmatrix} f + c\hat{n}G \\ \bar{r} - (f_r + c_r\hat{n}G) \end{bmatrix} + \frac{1}{k_j} Q^{-1}ae'_{n+j}Q^{-1} \begin{bmatrix} f + c\hat{n}G \\ \bar{r} - (f_r + c_r\hat{n}G) \end{bmatrix}$$

This indicates that the new solutions for x and $\hat{s}$ are divided into two parts: the solutions in the absence of the stated shortage of resource j and the impacts of the shortage of resource j.

The first part of the solution is already spelled out in Equations (49) and (50) in terms of $(I-A-c\hat{v}')^{-1}$. Since

$$(I-A-c\hat{v}')^{-1} = (I-A)^{-1} + Mzu'$$

by Equation (31), plugging this back into Equations (49) and (50) gives

$$Q^{-1} \begin{bmatrix} f+c\hat{n}G \\ \bar{r}-(f_r+c_r\hat{n}G) \end{bmatrix} = \begin{bmatrix} \{(I-A)^{-1} + Mzu'\}f + MzG \\ \bar{r} - r \end{bmatrix} \tag{55}$$

where

$$r = [B(I-A)^{-1} + M(Bz+c_r)u']f + f_r + M(Bz+c_r)G$$

as was given by Equation (34). The vector r represents total resource requirements to satisfy the given final demands f, f_r, and G in the absence of any limitations on resource availability.

Mathematical manipulation on the second part of Equation (54) leads to the following expressions (see Appendix):

$$e'_{n+j}\, Q^{-1} \begin{bmatrix} f+c\hat{n}G \\ \bar{r} - (f_r+c_r\hat{n}G) \end{bmatrix} = -(r_j - \bar{r}_j) \tag{56}$$

$$\frac{1}{k_j} Q^{-1} a e'_{n+j} Q^{-1} \begin{bmatrix} f+c\hat{n}G \\ r-(f_r+c_r\hat{n}G) \end{bmatrix} = \frac{-(r_j-\bar{r}_j)}{k_j} \begin{bmatrix} \{(I-A)^{-1}+Mzu'\}\hat{a} \\ -\{B(I-A)^{-1}+M(Bz+c_r)u'\}\hat{a}-e_j \end{bmatrix} \tag{57}$$

$$k_j = [B'_j(I-A)^{-1} + M(B'_j z+c_{rj})u']\hat{a} \tag{58}$$

where B'_j is the j-th row of B. On plugging Equation (57) into (54),

$$x = [(I-A)^{-1} + Mzu']f + MzG - [(I-A)^{-1}+Mzu'] \frac{(r_j-\bar{r}_j)\hat{a}}{k_j} \tag{59}$$

$$\hat{s} = \bar{r} - r + [B(I-A)^{-1}+M(Bz+c_r)u'] \frac{(r_j-\bar{r}_j)\hat{a}}{k_j} \tag{60}$$

$$x_d = \frac{(r_j-\bar{r}_j)}{k_j} \quad . \tag{60b}$$

The bracketed term, $B'_j(I-A)^{-1} + M(B'_j z+c_{rj})u'$, in Equation (58) stands for the j-th resource multiplier for f, indicating how much of resource j is required per unit of final outputs f. Therefore, k_j shows how much of resource j is tied up to the production reduction according to $\hat{a}$.

If $\hat{a} = e_i$, k_j represents the amount of resource j released by one unit reduction of production by sector i. If $\hat{a} = (\frac{1}{2}, \frac{1}{2}, 0\ldots,0)'$, k_j represents the amount of resource j saved jointly by a production reduction of ½ units in sector 1 and a production reduction of ½ units in sector 2. Since $(r_j-\bar{r}_j)$ is the total quantity of the shortage of resource j, $(r_j-\bar{r}_j)$ divided by k_j dictates how much in total the economy should initially cut down its production because of the shortage of resource j. As indicated in Equation (60b), $(r_j-\bar{r}_j)$ divided by k_j is equal to x_d, the total amount purchased by the dummy sector.

The total initial production reduction multiplied by $\hat{a}$,

$$\frac{(r_j-\bar{r}_j)\hat{a}}{k_j} \quad ,$$

accounts for how the initial reduction is actually carried out. For example, if the total reduction stands at \$300, i.e.,

$$\frac{(r_j-\bar{r}_j)}{k_j} = 300 \quad ,$$

and if $\hat{a} = e_i$, then sector i alone takes care of the total reduction by \$300. If $\hat{a} = (1/3, 1/3, 1/3, 0, \ldots, 0)'$, the reduction is divided up into \$100 for sector 1, 2, and 3, so that each of them slashes production by \$100. Therefore, $(r_j-\bar{r}_j)\hat{a}/k_j$ shows how the total initial production reduction is allocated among sectors.

Once the initial production reduction due to the resource shortage takes place, it touches off multiplier effects throughout the economy, because the initial reduction calls for, via interdependences among sectors of the economy, a series of production reductions by the other sectors, even though they were not initially hit by the resource shortage. The matrix preceded by $(r_j-\bar{r}_j)\hat{a}/k_j$ in Equation (59),

$$[(I-A)^{-1} + Mzu'] \; ,$$

accounts for such multiplier effects. Therefore, the third component with a negative sign in Equation (59) represents the total ultimate production reduction the economy undergoes as a result of the shortage of resource j. The first two components of Equation (59) represent, of course, economy's current production in the absence of such a shortage. Equation (59) may be rewritten as

$$x = x^o - [(I-A)^{-1} + Mzu'] \frac{(r_j-\bar{r}_j)\hat{a}}{k_j} \qquad (50')$$

where x^o is the input-output solution for x, i.e., the maximum production in the absence of any resource shortages.

Similarly, the third long component of Equation (60) accounts for the effect of the j-th resource shortage on employment of other resources. The bracketed term of it,

$$[B(I-A)^{-1} + M(Bz + c_r)u'] \; ,$$

stands for the resource-employment multiplier for f, indicating the amount of resources (other than resource j) required per unit of final outputs f. This term is non-negative and $(r_j-\bar{r}_j)\hat{a}/k_j$ is also non-negative. So, the whole third term is non-negative. Therefore, the third component measures a rise in resource unemployment due to the production reduction which is in turn caused by the shortage of resource j.

Thus, Equations (59) and (60) demonstrate the pervasive impacts of a shortage of a certain resource on the economy's production and employment of the resources under a certain objective-oriented production plan $\hat{a}$. What is left for explanation is the impact of the shortage on income and determination of the content of vector $\hat{a}$, which is discussed below.

Allocative decisions under single and multiple objectives and the shadow price equation

Since the objective function is stated as

$$Y = \hat{v}'x + \hat{n}G,$$

substituting x of Equation (59) into this objective function gives (see Appendix)

$$Y = Mu'f + MG - \frac{M(r_j - \bar{r}_j)u'}{k_j}\hat{a} \qquad (61)$$

By the assumption that there is a shortage in resource j, $r_j - \bar{r}_j > 0$. Also, M, u', k_j, $\hat{a} \geq 0$. If there is no such shortage (i.e., $r_j = \bar{r}_j$), the last term vanishes and the maximum income, Mu'f + MG, is guaranteed. So, the last term indicates a reduction in income due to the resource constraints. If we put $(r_j - \bar{r}_j) = 1$, the last term becomes

$$\frac{Mu'\hat{a}}{k_j} \; . \qquad (62)$$

This is none other than the shadow price of resource j, because it indicates the change in the value of the objective function (i.e., the change in income) that is achieved if the j-th resource constraint were tightened by one unit.

Equations (61) and (62) show that the level of income and the value of the shadow price resulting from the adjustment to a resource shortage depend on the size of the input-output Keynesian multiplier. The greater the multiplier, the greater the impact of the resource shortage on income and on the shadow price (the lower the resulting income and the higher the shadow price).

Equations (61) and (62) can be rewritten with k_j spelled out completely and with the shadow price denoted by P_s as follows:

$$Y = Mz'f + MG - \frac{M(r_j - \bar{r}_j)u'\hat{a}}{[B_j'(I-A)^{-1} + M(B_j'z + c_{rj})u']\,\hat{a}} \tag{61b}$$

$$P_s = \frac{Mu'\hat{a}}{[B_j'(I-A)^{-1} + M(B_j'z + c_{rj})u']\,\hat{a}} \ . \tag{62b}$$

Notice that vector $\hat{a}$ appears in both the numerator and denominator of both equations.

Suppose that the only objective is maximization of income, i.e., the efficiency objective. Since the last term in Equation (61b) has a negative sign, to achieve this objective requires this reduction of income due to the stated resource shortage to be minimized. Given that all the coefficients (technical coefficients, income coefficients, resource coefficients, disaggregated marginal propensities to consume) are constant and given the amount of the stated resource shortage, the problem then is how to choose vector $\hat{a}$ to achieve such an objective. The only constraint on vector $\hat{a}$ is that all components of $\hat{a}$ are non-negative and they add up to one. The mathematical programming problem to determine $\hat{a}$ is as follows:

$$\text{Min} \quad M(r_j - \bar{r}_j)u'\hat{a}/k'\hat{a}$$

$$\text{subject to} \qquad (63)$$

$$q'\,\hat{a} = 1$$

$$\hat{a} \geq 0$$

where

$$k' = B_j'(I-A)^{-1} + M(B_j'z + c_{rj})u'$$

$$q' = (1, 1, \ldots, 1)\ .$$

Called the fractional programming problem, programming problem (63) belongs to a class of non-linear programming. Consider the following two programming problems:

Problem I

$$\min\ (b'x + b_o)/(d'x + d_o)$$

subject to

$$A'x - c \geq 0$$

$$x \geq 0$$

where $d'x + d_o \geq 0$, b_o and d_o are scalars.

Problem II

$$\min\ b'Y + b_oY_o$$

subject to

$$A'Y - cY_o \geq 0$$

$$d'Y + d_oY_o = 1$$

$$Y \geq 0$$

$$Y_o > 0$$

where

$$h = d'x + d_o > 0 \tag{64}$$

$$Y_o = 1/h > 0 \tag{65}$$

$$Y = Y_ox \geq 0\ . \tag{66}$$

Charnes and Cooper (11) proved that Problem I and Problem II are equivalent, which can be summarized in the following theorem:

Theorem 3: Under the conditions of Equations (64) - (66), all feasible solutions in Problem I are feasible in Problem II, and conversely, except when $Y_o = 0$. Hence, if Y^* and Y_o^* are optimal solutions of Problem II, x^* is an optimal solution of Problem I, and $Y_o \neq 0$, then

$$\min\ (b'x + b_o)/(d'x + d_o) = b'Y^* + b_o Y_o^*$$

and $x^* = (1/Y_o^*)\ Y^*$ solves Problem I.

A use of Theorem 3 in our problem (63) leads to the following linear programming problem[1]:

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[1]Let $h = k'\hat{a}$, $w_o = 1/h$, and $w = w_o\hat{a} = \hat{a}/h$. Then,

$$M(r_j-\bar{r}_j)u'\hat{a}/k'\hat{a} = M(rj-\bar{r}_j)u'\hat{a}/h = M(r_j-\bar{r}_j)u'w.$$

On multiplying both sides of $q'\hat{a} = 1$ in Equation (63) by w_o,

$$q'w_o\hat{a} = w_o$$

which reduces to $q'w = w_o$. Since $w = w_o\hat{a}$, $k'w = w_o k'\hat{a}$. But $k'\hat{a} = h$ and $w_o = 1/h$. Hence, $w_o k'\hat{a} = 1$, so that $k'w = 1$. Let $\hat{a}^*$, w^*, and w_o^* be optimal solutions. Then by Theorem 3,

$$M(r_j-\bar{r}_j)u'\hat{a}^*/k'\hat{a}^* = M(r_j-\bar{r}_j)u'w^* .$$

Since $w_o^* = 1/h^*$, $M(r_j-\bar{r}_j)u'\hat{a}^*/k'\hat{a}^* = w_o^* M(r_j-\bar{r}_j)u'\hat{a}^*$. Therefore,

$$w_o^* M(r_j-\bar{r}_j)u'\hat{a}^* = M(r_j-\bar{r}_j)u'w^* \quad \text{or} \quad \hat{a}^* = w^*/w_o^* .$$

$$\text{Min } M(r_j - \bar{r}_j)u'w$$

subject to

$$q'w - w_o = 0 \;, \tag{63b}$$

$$k'w = 1 \;,$$

$$w \geq 0 \;,$$

$$w_o > 0 \;,$$

where w is a (nxl) vector and w_o is a scalar. This is a very simple linear programming problem with two constraints (besides the non-negative constraints) and with multipliers as coefficients. Theorem 3 says that solving this linear programming is equivalent to solving non-linear programming (63). Let $\hat{a}^*$, w^*, and w_o^* be optimal solutions. Then by Theorem 3,

$$\hat{a}^* = w^*/w_o^* \;. \tag{67}$$

Since the linear programming problem, Equation (63b), has only two constraints (except for non-negativity constraints), only two variables out of (n+l) variables enter the basis and, hence, only two variables have a positive value; w_o and one of w. Therefore, $\hat{a}^*$ is a unit vector, i.e., only one element of $\hat{a}^*$ is unity and all the other elements of $\hat{a}^*$ are zero. This implies that, to maximize income, only a single sector should be chosen for a production reduction to meet the stated resource shortage. Since $\hat{a}$ appears in both the numerator and denominator in Equation (61b), the sector to be chosen is the one whose coefficient in the objective function, $M(r_j - \bar{r}_j)u'/k'$, is the smallest.

Let sector i be chosen thereby. Then, $\hat{a}^* = e_i$. The maximum income and the associated shadow price under the production adjustment plan as indicated by $\hat{a}^* = e_i$, denoted by Y* and P_s^*, respectively, are given by

$$Y^* = Mu'f+MG-\{M(r_j-\bar{r}_j)u_i/[B_j'E_i+M(B_j'z+c_{rj})u_i]\} \qquad (61c)$$

and

$$P_s^* = Mu_i/[B_j'E_i+M(B_j'z+c_{rj})u_i] \qquad (62c)$$

where E_i is the i-th column of $(I-A)^{-1}$ and u_i is the i-th element of u (the income multiplier of sector i). The level of production and employment of resources that leads to this income and shadow price are obtained by substituting e_i for $\hat{a}$ in Equations (59) and (60).

A brief reference to Equation (34) shows that the denominator of Equations (61c) and (62c) stands for the requirement of resource j per unit of final output produced by sector i. Hence, the shadow price as given by Equation (62c) is in fact the productivity of resource j to the entire economy when used in sector i (not the productivity of resource j to sector i). The last term of Equation (61c) represents the amount of the 'ultimate' reduction in the economy's total income when one unit of the short-supplied resource is taken away from sector i.

What this implies is that, when maximization of income is the only one objective to be pursued, it is unnecessary to run a separate linear programming problem to determine vector $\hat{a}$. What is needed is simply to calculate, according to the formula given by Equation (62c), the productivity of resource j to the entire economy when used in each sector and choose the sector with the lowest productivity. If, as a result, sector k is chosen, set $\hat{a} = e_k$. The solutions for production and resource employment that result from setting $\hat{a} = e_k$ in Equations (59) and (60) are the ones that lead to the maximum income that can be achieved under the stated resource shortage.

If c = 0 and $\hat{c} = 0$, Equation (62c) reduces to

$$P_s = u_i / B'_j E_i \ . \tag{62d}$$

An economic evaluation of water by Lofting and McGauhey (44) reported this value for various industries in California. A similar study by Young and Martin (86) took this value as a criterion for water allocation in Arizona[1].

Suppose that there are several other objectives besides the efficiency objective to be considered in resource allocation decisions, because allocation of natural resources typically involves public interests. Since one cannot maximize every stated objective, the multiplicity of objectives involves conflicts among objectives and trade-off's among objectives (14, 20).

A solution which maximizes one objective will not necessarily maximize any of the other objectives. One important analytical goal of multi-objective analysis is to generate information that is presented to a decision maker in a manner that shows range of choices and trade-offs among objectives.

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[1]They called that value "income generating capacity" rather than the shadow price.

Suppose that we have two objectives denoted by Z_1 and Z_2 as shown in Figure 4. A closed set of choices formed by the polygon connecting points O, A, B, C, D, E, and F, constitutes the feasible region for choice-making in a hypothetical situation. By the northeast rule (14, p. 71), the non-inferior set of choices is found to be the heavy line connecting point B, C, D, and E. In other words, any points on this heavy line are not dominated by any other points in the feasible set, while any point in the feasible set which is not on the heavy line is dominated at least by one point on the heavy line. Hence, an optimal solution to be chosen must be on the heavy line, i.e., in the non-inferior set. It should be noted that solutions in the non-inferior set are not comparable. The amount of one objective that must be sacrificed to gain an increase in the other objective is called a trade-off (14, p. 74). A movement from one non-inferior solution to another non-inferior one involves such a trade-off. In Figure 4, the trade-off is measured by the slopes of the heavy line.

If the preference function is known, the "best-compromise" solution is determined at the point of the non-inferior set where the preference function and the configuration of the non-inferior set are tangent to each other. For instance, if a preference function is given by the curve p_1 as shown in Figure 4, the best-compromise solution is determined at point C. The articulation of the preference function belongs to the decision maker. One role of the analyst is to present such a configuration of the non-inferior set. But, the real problem is how to find out such a configuration.

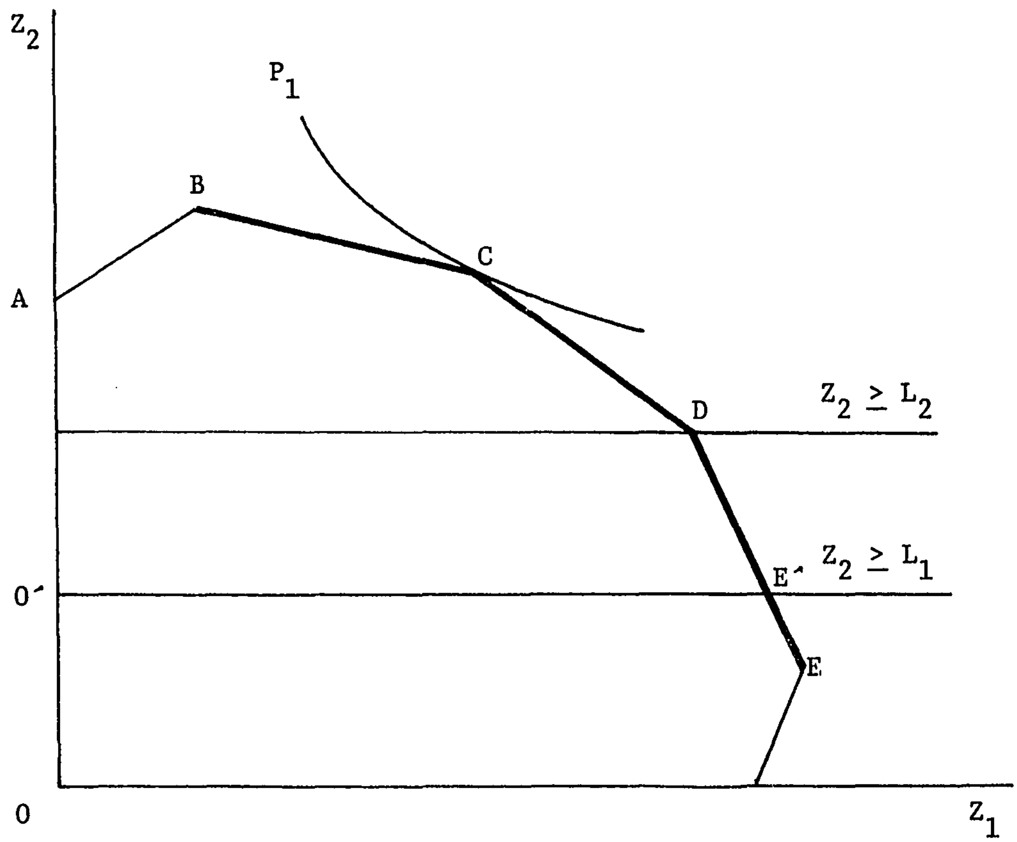

Figure 4. Illustration of the constraint method in generating the configuration of the noninferior set of choices in a two-objective case.

The most intuitively appealing technique to generate the contour of the non-inferior set is what is known as the constraint method (14, p. 115; 34, p. 223). The method essentially involves imposing a constraint (or constraints) on one objective (or objectives where there exist more than two objectives) and solving the constrained optimization problem by successively changing the allowable level of the other objective(s).

Suppose that, in Figure 4, Z_1 is the efficiency objective (maximization of Y in our model) and that $Z_2 \geq L_i$ where L_i is the minimum allowable level of Z_2. Depending on different values of L_i, two different horizontal lines are drawn in Figure 4. Notice that, in Figure 4, Z_1 is maximized at point E in a separate maximization of Z_1. In our problem of choosing $\hat{a}$ in Equation (63) (or w in Equation 63b), point E corresponds to the solution derived from substituting $\hat{a}^* = e_i$ as obtained in problem (63) or (63b) into objective function (61b). Likewise, a separate maximization of Z_2 gives point B as an achievable maximum of Z_2. Therefore, two separate maximizations of Z_1 and Z_2 provide two points in objective space, B and E, to be initially observed. The constraint, $Z_2 \geq L_1$, is shown to reduce the feasible region from the original polygon OABCDEF to the new polygon O'ABCDE', so that the original income maximization point, E, is not feasible any more. Point E' is a new observed point in the contour of the non-inferior set. An important observation is that Z_1 is now maximized at E' in the new feasible set. Therefore, E' can be found by maximizing Z_1 subject to $Z_2 \geq L_1$ (and, of course, subject to the other constraints associated with decision variables involved in the model). Similarly, another point of the non-inferior set is found by maximizing Z_1 subject to $Z_2 \geq L_2$ and other constraints, which gives point D to be observed in objective space. By successively changing the value of L_i, the configuration of the non-inferior set can be brought to the surface.

In applying the constraint method to our model, the problem under the stated resource shortage is now to choose, subject to the considera-tion of additional objectives, a production reduction plan embodied in vector $\hat{a}$ that leads to non-inferior solutions. Since a maximum income is attained by minimizing the last term of Equation (61b), the programming problem (63) can be revised as follows:

$$\begin{aligned} &\text{Min } M(r_j - \bar{r}_j)u'\hat{a}/k'\hat{a} \\ &\text{subject to} \\ &\quad q'\hat{a} = 1 \\ &\quad H\hat{a} \geq h \\ &\quad \hat{a} \geq 0 \end{aligned} \qquad (63c)$$

where the second set of constraints, $H\hat{a} \geq h$, reflects constraints on $\hat{a}$ due to the consideration of objectives other than maximization of income. Let the resulting solution be $\hat{a}^{**}$. Possibly, $\hat{a}^* \neq \hat{a}^{**}$. If $\hat{a}^* = \hat{a}^{**}$, this implies that the second set of constraints are redundant, i.e., they are not binding in attaining the maximum income under the single objective of maximization of income. If this second set of constraints is effectively binding, $\hat{a}* \neq \hat{a}**$. Since production adjustment to the resource shortage under $\hat{a}*$ leads to a maximum level of income, the production adjustment under $\hat{a}**$ necessarily leads to a lower level of income. Therefore, the difference between the level of income achieved under $\hat{a}*$ and under $\hat{a}**$ reflects how much income should be forgone to satisfy other stated non-efficiency objectives, i.e., the imputed costs associated with non-efficiency objectives. For instance, suppose that $\hat{a}**$ resulted from adding the set of constraints associated with a consideration of income distribution to the efficiency objective. Then, the difference between the level of income under $\hat{a}*$ and under $\hat{a}**$ measures the trade-off ratio between the efficiency objective and the income distribution objective. Similarly, if a consideration of national security, in addition to the efficiency objective, lead to $\hat{a}**$, then the difference between the income level under $\hat{a}*$ and under $\hat{a}**$ indicates the trade-off between national security and income.

The solution for $\hat{a}$ obtained from programming problem (63c) is always relative to the constraints imposed on $\hat{a}$. A different set of the constraints on $\hat{a}$ would lead to a different $\hat{a}$ and to a different set of trade-offs. One thing most remarkable about the constraint method as applied to our model is that, since the impact of the stated resource shortage is collected into a separate term in the objective function as stated in Equation (61b), it is unnecessary to solve the whole linear programming problem (51) integrating the constraints on x, s, and $\hat{a}$ (actually x_{id}'s) in order to obtain an optimal $\hat{a}$ or to derive trade-offs among objectives. What is required is simply to look at the objective function as spelled out in Equation (61b) and to set up a separate reduced program which includes only the constraints on the vector $\hat{a}$. Thereby, the solution process for an optimal $\hat{a}$ and the generation and evaluation of alternatives in terms of several objectives are greatly simplified. In particular, the single objective of income maximization dispenses with even such a separate program.

The input-output Keynesian multiplier modified by a resource shortage

Equation (61) is reproduced here for an easy reference:

$$Y = Mu'f + MG - Mu'(r_j - \bar{r}_j)\hat{a}/k_j \ . \qquad (61)$$

Once a shortage of resource j takes place by the amount of $(r_j - \bar{r}_j)$, production of the economy should be trimmed off by the amount of

$$(r_j - \bar{r}_j)\hat{a}/k_j \ .$$

This production reduction has a multiplier effect on income, with the (vector) multiplier given by Mu'. As a result, the total income of the economy reduces by the amount indicated by the last term with a negative sign in Equation (61). It is clear that the greater the shortage, the larger the magnitude of production re-adjustment, and the greater the reduction in income. With the total supply of resource j given by $\bar{r}_j$, the magnitude of the shortage is determined by the demand for the resource, because

$$r_j = [B_j'(I-A)^{-1} + M(B_j'z+c_{rj})u']f + M(B_j'z+c_{rj})G + f_{rj} \quad . \tag{68}$$

In other words, the magnitude of the shortage depends on the level of autonomous spending f and G. An increase in autonomous spending is shown to augment the shortage. So, a change in the level of autonomous spending has a two-pronged effect on income in the presence of a resource shortage. On the one hand, an increase in autonomous spending increases income via the multiplier process. On the other hand, it adds to the total resource requirement of the economy and, after output substitution taking place as dictated by the vector $\hat{a}$, causes a reduction in income. On balance, the income increasing effect of the increase in autonomous spending is partially choked off by its income decreasing effect.

Substituting Equation (68) into Equation (61) and rearranging terms with respect to exogenous variables f, G, and f_{rj} (see Appendix),

$$Y = \hat{M}u'f + \hat{M}G + \frac{\hat{M}u'\hat{a}}{B'_j(I-A)^{-1}\hat{a}} [\bar{r}_j - B_j'(I-A)^{-1}f - f_{rj}], \tag{61d}$$

or alternatively

$$Y = \hat{M}u'f + \hat{M}G + \frac{1}{B_j'z+c_{rj}} (1 - \frac{\hat{M}}{M}) [\bar{r}_j - B_j'(I-A)^{-1}f - f_{rj}], \tag{61e}$$

where

$$\hat{M} = \frac{1}{1 - (u'c+\hat{c}) + (B_j'z+c_{rj})u'\hat{a}/B_j'(I-A)^{-1}\hat{a}} \quad . \tag{69}$$

Equations (61d) and (61e) bring forth a new input-output Keynesian multiplier $\hat{M}$ which is modified by the resource shortage. Therefore, a one-dollar increase in non-commodity spending G creates additional income not by M dollars, but by $\hat{M}$ dollars now.

What makes a difference between this new multiplier and the original input-output Keynesian multiplier M is the last term in the denominator of $\hat{M}$. This last term can be rewritten as

$$u' \frac{(B_j' z + c_{rj})\hat{a}}{B_j'(I-A)^{-1}\hat{a}} \quad . \tag{69b}$$

The denominator of the fraction refers to the requirement of resource j associated with $\hat{a}$ in the absence of the income effect of production and the numerator to the requirement of resource j solely arising from the income effect. So, the fraction reflects the relative significance of the income effect on resource j (relative to the resource requirement in the absence of such effect). Expression (69b) represents the portion of the change in income associated with $\hat{a}$ that is explained by the income effect. If there is no such income effect on resource j (or, if it exists, it is negligibly small), expression (69b) vanishes and

$$\hat{M} = M \ .$$

Also, if there is no shortage with respect to resource j (i.e., $\hat{a} = 0$), then expression (69b) vanishes and again $\hat{M}$ coincides with M. Since expression (69b) is supposed to be non-negative,

$$\hat{M} \leq M \ ,$$

implying that the resource shortage dampens the magnitude of the input-output Keynesian multiplier. The extent to which the multiplier effect dampens depends first on which sector(s) is (are) selected for production adjustment (i.e., depends on $\hat{a}$). After $\hat{a}$ is determined, it then depends on the requirements of resource j by the sector(s), on the significance of the income effect on resource requirements of the sector(s), and on the magnitude of the multiplier u of the sector(s). So, the greater the income effect on the use of resource j, the smaller the resource requirement, the greater the multiplier, the greater will the dampening effect be.

Equations (61d) and (61e) demonstrate the above-mentioned conflicting effect of commodity spending f on income. A one-dollar increase in f will raise the level of income by Mu'f dollars initially, but the economy-wide output substitution process following the resulting increased shortage will exert a downward pressure on income measured by

$$\frac{B_j'(I-A)^{-1}}{B_j'z+c_{rj}}\left(1-\frac{\hat{M}}{M}\right) .$$

So, the net effect turns out to be a compromise of these conflicting forces.

CHAPTER V. SUMMARY, CONCLUSIONS, AND FURTHER RESEARCH NEEDS

Summary and Conclusions

The first objective of this study sought to develop an analytical framework integrating both competitive and interdependent dimensions of resource uses. To achieve this objective, a linear programming model that consists of an input-output system and resource constraints, was formulated.

The second objective was to apply the model to Northwest Iowa. For an application of the model, the 12-county area in Northwest Iowa was selected. The application was geared to determining whether or not the water resources of Northwest Iowa can support the region's population and economic growth as projected by the State of Iowa to the year 2020, given the current water use rates and given the current production structure and inter-sectoral relations as embodied in the input-output table of Iowa. For the input-output system, the 13-industry input-output table of Iowa was adopted (4). Seven water supply sources were identified for the region; three ground water sources and four surface water sources. Under the assumption that water supply sources of each county are independent of each other and each county's water availability from a particular water supply source is not augmented by water transportation, an individual water supply constraint was imposed on each supply source of each county.

Because of the difficulty in predicting the future of irrigation in Northwest Iowa, despite an expectation of a substantial increase in irrigation particularly in this region, some upper limits to irrigation were established in terms of land and water availability for irrigation.

The application began with an analysis of data on the region's population and economic status and on its growth projections. Based on the results of the analysis, the level of final demands to support projected population and economic growth of the region was specified. This was followed by estimation of production and water requirements to satisfy the specified final demand goal. The water coefficients used in estimation of total water requirement were taken from the water study by Barnard and Dent (5). However, the water coefficient of irrigated corn production was independently estimated county by county for this study.

The results of the application show that the level of final demands that can be achieved subject to the water supply constraints of Northwest Iowa coincided with the target level of final demands that was imposed in the model, implying that the available water supplies of Northwest Iowa do not constitute a limiting factor to achieving the target level of final demands which was based on income and population growth projected for the region to the year 2020.

Irrigation was shown to be by far the largest water consuming activity. Should all the Class I and II land of the region be irrigated, more than 85 percent of the total water requirement of the region would go to irrigation, leaving only 15 percent of the total to be shared among industrial and final uses. Even in this case, the region's total water requirement falls short of the amount of water that can be supplied from the surficial aquifer of the Missouri River flood plain, setting aside the other surficial aquifers.

Hence, it can be concluded that Northwest Iowa as a whole holds potentially sufficient water supplies to depend on for the region's population and economic growth, which is not much out of line with the projection series made by the State of Iowa.

In the subregional level, however, the substantial water requirement associated with expansion of irrigation may impose a heavy burden on the subregion's water resources. To irrigate both Class I and II land, most counties of the region must reach out to reservoir storage, the most expensive water supply source in the region, for additional water. Some counties (Buena Vista, Clay, O'Brien, Osceola, Sac, and Sioux) may not be able to irrigate all their Class I and II land due to water shortages. Irrigation of Class I and II land was shown to push shadow prices associated with water supply constraints of bedrock aquifer, surficial aquifers, interior streams, and Big Sioux River to positive levels, $1.00, $1.50, $1.30, and $1.30 per 1,000 gallons, respectively.

Due to different water availability, expansion of irrigation forces some counties of the region to exhaust cheap water supply sources more quickly than the other counties and to turn toward more expensive water supply sources. Notably, even if all of its Class I and II land be irrigated, Woodbury County on the Missouri River flood plain has a tremendous water surplus. Furthermore, the surplus consists of the cheapest water in the region. This suggests desirability of water transfers from surplus areas to shortage areas. Since the surficial aquifer of the Missouri River flood plain constitutes the largest, as well as the cheapest, water supply source in the region, it should come first under consideration of the region-wide water distribution. However, such water distribution may claim a huge amount of outlays (64). Therefore, a careful evaluation of costs and benefits is required to determine the scale of the region-wide water transfers.

The third objective of this study was to suggest an extended model for the future application to state and regions. The extension was necessary to obtain more accurate estimates of production and resource requirements incorporating the effect of an increase in income on production and resource uses and also to allow an impact analysis incorporating multipliers and allocative decisions under possible multiple objectives. Comprising the open input-output system, the applied model may significantly under-estimate production and resource requirements, the level of income and, hence, shadow prices.

Development of the extended model began with an extension of the basic input-output system of the applied model. The result of this extension shows that the difference between the estimate without and with the income effect involves the Keynesian multiplier as derived from the input-output system, indicating an importance of estimation of the Keynesian multiplier in estimation of production and resource requirement. A comparison of the multiplier process of the input-output system with that of the orthodox Keynesian macro-model shows that under certain assumptions, the Keynesian multiplier of the orthodox Keynesian macro-model can be substituted for the Keynesian multiplier as derived from the input-output system.

Combining the extended input-output system with the resource constraints produced the extended linear programming model that incorporates the effect of income on production and resource uses. Based on the resulting model, an impact analysis was presented, i.e., an analysis on the impact effects of a change in demand and supply of a certain resource on production, resource uses, and income. The analysis provided a short-cut method to measure such impacts, including the formula of the Keynesian multiplier modified by a resource shortage and the formula of the shadow price expressed as a function of the Keynesian multiplier and allocative decisions. The analysis described also how the multiplicity of objectives affects allocation decisions, thereby providing a short-cut method to calculate trade-off's among multiple objectives.

Application of the extended linear programming model and the results of the impact analysis requires additional information on the income consumption linkage in particular. Application procedures are briefed below.

Extension of Current Results to the Region and State

The extended model was expressed in such a way as to retain the basic structure of the original applied model as much as possible. As a result, in applying the extended model to Northwest Iowa to obtain improved results, what is needed is simply to modify the technical coefficient matrix by incorporating the commodity-wise consumption data into it. If data on the marginal propensities to consume resources are available, the resource requirement coefficient matrix (B matrix in the applied model) needs to be modified to incorporate such data. Since the extended model differentiates commodity expenditures (autonomous consumption expenditures on produced goods and services) and non-commodity expenditures (autonomous consumption expenditures on resources), data on final demand need to be compiled according to the differentiation. Once all these data have been lined up, a re-run of the model would provide a new set of estimates of production and water requirements and shadow prices incorporating the income effect.

Since the technical coefficient matrix, the resource requirement coefficient matrix (including the water coefficients), and commodity-wise consumption data are available for the state, the extended model is directly applicable to the state, when detailed state-wide water supply data become available. Since detailed data on final demands of the state are already available from the input-output study by Barnard (4), the tedious procedures of estimating final demand as followed in this study are waived.

The water coefficient of each industrial sector of the state was already estimated by Barnard and Dent (5). However, the water coefficient of crop production estimated by them is based on the insignificant irrigation practice in Iowa. Therefore, it needs to be revised to reflect the expected increase in irrigation in Iowa. The water coefficient of crop production estimated for Northwest Iowa can be extended to the state-wide application. Alternatively, the water coefficient of crop production in the other regions of Iowa can be independently estimated following the same procedures as used in this study and then combined with the water coefficients of crop production estimated for Northwest Iowa.

In applying the extended model to the state, the technical coefficient matrix made by Barnard (4) should be modified to incorporate the commodity-wise consumption data. The resource requirement matrix also can be modified to incorporate the marginal propensities to consume resources, if such data are available.

Except for these minor modifications, the procedures of applying the extended model to the state are basically the same as those followed in this study, because the Northwest Iowa economy was treated as a miniature of the entire Iowa economy in this study's application. However, selections of procedures to be followed and kinds of data to be used totally depend on the nature of the future research.

Further Research Needs

In the input-output framework, all the costs of economic activities should be addressed in terms of intermediate and resource inputs. Due to inadequate data, the application of this study put the monetary costs of water supply directly into the objective function. However, a proper treatment of water supply costs requires the costs to be estimated in terms of intermediate and resource inputs and to be incorporated into the input-output system rather than into the objective function. This implies that the input-output system be expanded to comprise water production as a separate sector (or sectors depending on the number of different water supply sources).

In the application of the model, water transportation was assumed away. However, the results of the application indicate desirability of water transfers. Therefore, a model integrating water transportation activities needs to be developed. The water transportation costs should be treated within the input-output framework like the costs of other activities.

This study was concerned with the quantity dimension of the water problem, ignoring the water quality dimension. A supply of water with a particular quality may serve a number of purposes unequally well. Different uses demand different properties in water or at least vary in their toleration of particular properties. Water can be regarded as differentiated in kinds and grades determined by its quantity. Thus, supply and demand functions of water can be each regarded as consisting of numerous quality-oriented segments, each segment characterized by relatively homogeneous quality (80, p.7). In connection with the model applied in this study, this implies that demand for and supply of water need to be further subdivided according to water quality variations. The water supply constraints are then to be specified in terms of both quantity and quality.

When the water quality dimension is introduced, the competition among water uses over limited water supplies may involve both quantity and quality elements. One use of water may reduce availability of water for the next uses, not only quantitatively, but also qualitatively by lowering water quality. Thus, one user of water may be in a position to retain benefits from use while shifting costs to other users by lowering water quality. This condition is termed externalities (80, p. 10). There have been quite a few studies that deal with this externality problem in the input-output framework (30, 43, 46, 72). Hence, further studies are needed in order to integrate input-output analysis on externalities into the linear programming model as formulated in this study.

The assumption underlying the application in this study is that all the coefficients involved, water coefficients in particular, be invariant over time. This may not be the case. Rising water supply costs may discourage waste in water uses and encourage more recirculation of water. The implementation of water pollution control may prompt water saving technology. However, technology may not always be water saving. New production processes may increase water use rates if the new production technique makes cost saving in other inputs sufficient to cover the increased water use. Therefore, the future research should involve reasonable projections of technical and water coefficients based on the future trends.

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APPENDIX: MATHEMATICAL FOOTNOTE

Let $(I-A)^{-1} = E$. By Theorem 2,

$$(I-A-c\hat{v}')^{-1} = E + \frac{Ec\hat{v}'E}{1-\hat{v}'Ec} = E + \frac{\hat{n}Ecv'E}{1-\hat{n}v'Ec}$$

$$= E + \frac{Ecv'E}{1-\hat{c}-v'Ec} \quad (\text{because } \hat{n} = \frac{1}{1-\hat{c}})$$

$$= E + Mzu' .$$

That is,

$$(I-A-c\hat{v}')^{-1} = E + Mzu' . \qquad (F1)$$

Using this result, we can obtain the following expressions:

$$\hat{v}'(I-A-c\hat{v}')^{-1} = \hat{v}'(E + \frac{Ec\hat{v}'E}{1-\hat{v}'Ec}) = \frac{\hat{v}'E}{1-\hat{v}'Ec}$$

$$= \frac{\hat{n}v'E}{1-\hat{n}v'Ec} = \frac{u'}{1-(u'c+\hat{c})} = Mu' ; \qquad (F2)$$

$$(I-A-c\hat{v}')^{-1}c = (E + \frac{Ec\hat{v}'E}{1-\hat{v}'Ec})c = \frac{Ec}{1-\hat{v}'Ec} ; \qquad (F3)$$

$$(B+c_r\hat{v}')(I-A-cv')^{-1} = B[E + \frac{zu'}{1-(u'c+\hat{c})}] + \frac{c_r u'}{1-(u'c+\hat{c})} \quad (\text{by F1 and F2})$$

$$= BE + \frac{(Bz+c_r)u'}{1-(u'c+\hat{c})} = BE + M(Bz+c_r)u' ; \qquad (F4)$$

$$1 + \hat{v}'(I-A-c\hat{v}')^{-1}c = 1 + \frac{\hat{v}Ec}{1-\hat{v}'Ec} \quad \text{(by F3)}$$

$$= \frac{1}{1-\hat{v}'Ec} \; ; \quad \text{and} \tag{F5}$$

$$c_r + (B+c_r\hat{v}')(I-A-c\hat{v}')^{-1}c$$

$$= B(I-A-c\hat{v}')c + c_r[1+\hat{v}'(I-A-c\hat{v}')^{-1}c]$$

$$= \frac{BEc+c_r}{1-\hat{v}'Ec} \quad \text{(by F3 and F5)}$$

$$= \frac{Bz+c_r}{1-\hat{v}'Ec} \quad . \tag{F6}$$

In Equation (54),

$$Q^{-1} \begin{pmatrix} f + c\hat{n}G \\ \bar{r} - (f_r+c_r\hat{n}G) \end{pmatrix}$$

$$= \left(\begin{array}{c:c} (I-A-c\hat{v}')^{-1} & 0 \\ \hdashline -(B+c_r\hat{v}')(I-A-c\hat{v}')^{-1} & I_m \end{array}\right) \begin{pmatrix} f + c\hat{n}G \\ \bar{r} - (f_r+c_r\hat{n}G) \end{pmatrix} .$$

By Equation (F3),

$$(I-A-c\hat{v})^{-1} \; c\hat{n}G = \frac{\hat{n}EcG}{1-\hat{v}'Ec} = MzG \quad .$$

By Equation (F6),

$$[(B + c_r\hat{v}')(I-A-c\hat{v}')^{-1}c + c_r]\hat{n}G = \frac{\hat{n}(Bz+c_r)}{1-\hat{v}'Ec} G$$

$$= M(Bz+c_r)G \ .$$

By Equation (F4),

$$(B + c_r\hat{v}')(I-A-c\hat{v}')^{-1}f = BE + M(Bz+c_r)u' \ .$$

Hence,

$$Q^{-1}\begin{bmatrix} f + c\hat{n}G \\ \bar{r} - (f_r + c_r\hat{n}G) \end{bmatrix} = \begin{bmatrix} (E + Mzu')f + MzG \\ \bar{r} - r \end{bmatrix} \tag{F7}$$

where

$$r = [BE + M(Bz+c_r)u']f + M(Bz+c_r)G + f_r \ . \tag{F8}$$

From Equation (F7),

$$e'_{n+j}\ Q^{-1}\begin{bmatrix} f + c\hat{n}G \\ \bar{r} - (f_r + c_r\hat{n}G) \end{bmatrix} = \bar{r}_j - r_j \ . \tag{F9}$$

Since

$$Q^{-1} = \left[\begin{array}{c|c} E + Mzu' & 0 \\ \hline -\{BE + M(Bz+c_r)u'\} & I_m \end{array}\right] ,$$

$$Q^{-1}a = Q^{-1} = \begin{bmatrix} \hat{a} \\ -e_j \end{bmatrix} = \begin{bmatrix} (E + Mzu')\hat{a} \\ -\{BE + M(Bz+c_r)u'\}\hat{a} - e_j \end{bmatrix} , \tag{F10}$$

where e_j is a (mx1) vector whose j-th element is one and all other elements are zero. From Equation (F10),

$$\frac{1}{k_j} Q^{-1} a\, e'_{n+j} Q^{-1} \begin{bmatrix} f + cnG \\ \bar{r} - (f_r + c_r \hat{n} G) \end{bmatrix}$$

$$= \frac{\bar{r}_j - r_j}{k_j} \begin{bmatrix} (E + Mzu')\hat{a} \\ -\{BE + M(Bz+c_r)u'\}\hat{a} - e_j \end{bmatrix} \tag{F11}$$

Since,

$$1 + e'_{n+j} Q^{-1} a = e'_{n+j} \begin{bmatrix} (E + Mzu')\hat{a} \\ -\{BE + M(Bz+c_r)u'\}\hat{a} - e_j \end{bmatrix} +1$$

$$= -\{B'_j E + M(B'_j z + c_{rj})u'\}\hat{a} \ ,$$

$$k_j = -(1 + e'_{n+j} Q^{-1} a) = [B'_j E + M(B'_j z + c_{rj})u']\hat{a} \ . \tag{F12}$$

Combining Equations (F7) and (F11) gives Equations (59) and (60). Notice that in Equation (60) $\hat{s}$ excludes the j-th resource.

Since $x_d = e'_j \hat{s}$,

$$x_d = \bar{r}_j - r_j - \frac{\bar{r}_j - r_j}{k_j} \{[B'_j E + M(B'_j z + c_{rj})u']\hat{a} - 1\} \ .$$

Substituting Equation (F12) into this expression gives

$$x_d = \frac{r_j - \bar{r}_j}{k_j} \ .$$

Since $Y = \hat{v}'x + \hat{n}G$, substituting x of Equation (59) leads to

$$Y = \hat{v}' \quad \{(E + Mzu')\ [f - \frac{(r_j - \bar{r}_j)}{k_j}\ \hat{a}] + MzG\} + \hat{n}G$$

$$= \hat{n}\ (1+Mv'z)\ \{u'[f - \frac{(r_j - \bar{r}_j)}{k_j}\ \hat{a}] + G\ \} \quad .$$

Since

$$M = \frac{\hat{n}}{1-\hat{v}'z} \quad , \qquad \hat{n} = \frac{M}{1+Mv'z} \quad .$$

Hence,

$$Y = M\ \{u'[f - \frac{(r_j - \bar{r}_j)}{k_j}\ \hat{a}] + G\ \}$$

$$= Mu'f + MG - \frac{M(r_j - \bar{r}_j)u'}{k_j}\ \hat{a} \quad , \tag{F13}$$

which is Equation (61). Substituting r of Equation (F8) and k_j of Equation (F12) into Equation (F13) provides

$$Y = \frac{M}{k_j}\ \{[B_j'E + M(B_j'z+c_{rj})u']\hat{a}(u'f) + [B_j'E + M(B_j'z+c_{rj})u']\hat{a}G$$

$$- r_ju'\hat{a} + \bar{r}_ju'\hat{a}\ \}$$

$$= \frac{M}{k_j}\ \{\ (B_j'E\hat{a})u'f - (B_j'Ef)(u'\hat{a}) + (B_j'E\hat{a})G - (u'\hat{a})f_{rj} + (u'\hat{a})\bar{r}_j\ \} \ .$$

Let

$$\frac{M}{k_j} B_j'E\hat{a} \; [= \frac{M(B_j'E\hat{a})}{B_j'E\hat{a} + M(B_j'z+c_{rj})(u'\hat{a})}$$

$$= \frac{1}{1-(u'c+\hat{c}) + (B_j'z+c_{rj})u'\hat{a}/B_j'E\hat{a}}] \equiv \hat{M} \quad . \tag{F14}$$

Then,

$$\frac{M(u'\hat{a})}{k_j} = \frac{(u'\hat{a})/B_j'E\hat{a}}{1/M + (B_j'z+c_{rj})u'\hat{a}/B_j'E\hat{a}} = Mu'\hat{a}/B_j'E\hat{a} \quad . \tag{F15}$$

On substituting Equations (F14) and (F15) into Equation (F13), Equation (61c) is obtained.